

VATHEMATICS

ARNING FACILITATOR'S MANUAL





Mathematics 9

LEARNING FACILITATOR'S MANUAL





NOTE: This Mathematics 9 Learning Facilitator's Manual contains the Final Test and answers to the Final Test and to the teacher-assessed assignments; therefore, teachers should at all times keep it secure against unauthorized student access. The Final Test should be accessible to students only in supervised situations.

Mathematics 9 Learning Facilitator's Manual Modules 1–6 Learning Technologies Branch ISBN 0-7741-1098-8

This document is intend	led for
Students	
Teachers	-
Administrators	
Parents	
General Public	
Other	



You may find the following Internet sites useful:

- · Alberta Education, http://ednet.edc.gov.ab.ca
- · Learning Technologies Branch, http://ednet.edc.gov.ab.ca/ltb
- Learning Resources Distributing Centre, http://ednet.edc.gov.ab.ca/lrdc

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Teachers

Register with the Learning Technologies Branch

The Learning Technologies Branch is dedicated to upgrading and continually improving your Learning Facilitator's Manual so that it accurately reflects any necessary revisions we have had to make in the Student Module Booklets, Assignment Booklets, or the sample Final Test. The types of revisions that will be made are those that make the course more accurate, more current, or more effective.

LTB will send you the **latest enhancements** or **minor upgrades** for your Learning Facilitator's Manual if you return the following registration card to: Learning Technologies Branch, Box 4000, Barrhead, Alberta T7N 1P4, Attention: Instructional Design and Development.

	ng Facilitator's Manual pistration Card
First Name	Surname
School Name	School Phone Number
School Address	
City	Postal Code
Course Title	Approximate Date of Purchase



You can help ensure that distance learning courseware is of top quality by letting us know of areas that need to be adjusted. Call the Learning Technologies Branch, free of charge by dialling 310-0000, and ask for the Editing Unit. Also, a teacher questionnaire has been included at the back of most Learning Facilitator's Manuals. Please take a moment to fill it out.

We look forward to hearing from you!

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Introduction

A survey of these course materials will confirm that this learning package has been specially designed for many kinds of teachers working in a variety of situations.

In Which Category Do You Fit?

- ☐ Small Schools Teacher
 - inexperienced
 - a experienced, but in other subject areas
 - a experienced in teaching Mathematics 9, but wanting to try a different approach
- ☐ Distance Learning Teacher
 - ☐ travelling to schools within the region
 - using facsimile and teleconferences to teach students within the region
- ☐ Large Schools Teacher
 - inexperienced
 - a experienced in teaching Mathematics 9, but wanting to try a different approach



Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers, regardless of the situation.

Advantages for Students

Materials

- incorporate a strong learner-centred philosophy
- promote such qualities in the learner as autonomy, independence, and flexibility
- are developed through media that suit the needs and circumstances of the learner
- reflect the experiential background of Alberta students
- provide opportunities by overcoming barriers that result from geographical location
- promote individualized learning, allowing learners to work at their own pace

Advantages for Teachers

Materials

- allow teachers maximum teaching time and minimize preparation time
- include different routes through the materials to suit different learners
- incorporate a wide range of teaching strategies, in particular those using independent and individual learning
- deliver curriculum designed by education specialists that reflects the Alberta Education Program of Studies with an emphasis on Canadian content
- provide learning materials that are upwardly compatible with advanced educational technology

Does this learning package sound like something you could use?

This Learning Facilitator's Manual begins with an overview of the current Alberta Education Program of Studies for Mathematics 9. This summary is included for inexperienced teachers or those teachers who have found themselves teaching Mathematics 9 when their training is in other subject areas. This brief description is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this learning package and determine how they might want to use it in their classrooms.

Beyond the introduction, the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The Student Module Booklets, Assignment Booklets, and LFMs are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.



Overview of the Program of Studies

The distance learning materials for Mathematics 9 reflect the beliefs, goals, and expected student outcomes of the document *Common Curriculum Framework for K-12 Mathematics* (June 1995), which provides the basis for the programs of study in Alberta, British Columbia, Manitoba, Saskatchewan, Northwest Territories, and the Yukon. An overview of the Common Curriculum is provided here for your convenience; however, it is recommended that you obtain a copy of this document.

Beliefs About Students and Mathematics Learning

The Common Curriculum reaffirms the belief that students are active learners who have had varying experiences and who hold different views about mathematics and life. Students need to explore, reason, communicate mathematical ideas, and construct their own meaning of mathematics. Students need numerous and varied experiences in order to appreciate the usefulness of mathematics and to view mathematics as an integrated whole. Moreover, it is important that learning activities proceed from the concrete to the abstract and from the simple to the complex.

Goals for Students

The collaborating provinces and territories agree that mathematics education must prepare students to solve problems, communicate and reason mathematically, and appreciate and value mathematics. In order to function effectively in this technological age, students must become familiar with calculators and computers. It is important that students develop a positive attitude toward mathematics and have a base of knowledge and skills related to all aspects of mathematics.

Expected Student Outcomes

The content of the Common Curriculum is stated in terms of outcomes. These outcomes are measurable and identify what the students are expected to know and do. The outcomes for the entire Kindergarten to Grade 12 framework are organized within four strands: Number, Patterns and Relations, Shape and Space, and Statistics and Probability. The student outcomes incorporate seven mathematical processes: communication, connections, estimation and mental math, technology, problem solving, reasoning, and visualization. The outcomes are also influenced by the nature of mathematics and such recurring themes as shape, dimension (size and scale), number, quantity, pattern, relationships, uncertainty, constancy, and change.

Suggested Time Allotments

Considerable time should be spent on the concepts and processes identified within each strand. The chart on the right outlines the suggested time allotments for mathematics in grades 5 to 9.

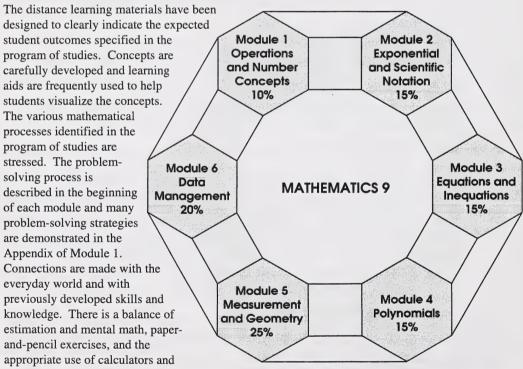
Mathematics in Grades 5 to 9						
Percent of Time Percent of Time						
Number Patterns and Relations Shape and Space Statistics and Probability	25–35 20–30 20–30 15–25					

The time allotment will vary from grade to grade. For example, a lower percentage of time will be spent on the number strand in Mathematics 9 than in Mathematics 7, and a higher percentage of time will be spent on Patterns and Relations in Mathematics 9 than in Mathematics 8.

The Common Curriculum emphasizes that concepts need to be developed gradually; concepts should be introduced using manipulatives, and developed from the concrete to the pictorial to the symbolic. Moreover, a minimum of half of the available time within all strands needs to be dedicated to the mathematical processes of communication, connections, estimation and mental math, technology, problem solving, reasoning, and visualization. Further, there should be a balance between estimation and mental math, paper-and-pencil exercises, and the appropriate use of calculators and computers.

Overview of Mathematics 9

Mathematics 9 contains six modules. The percentage appearing in each box suggests the approximate proportion of work time you should devote to that module.



computers. Students are given opportunities to communicate and reason mathematically. Themes such as pattern, relationships, constancy, and change are explored within the modules.

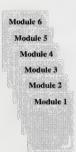
Every attempt has been made to give students a variety of activities and to make mathematics real and interesting. Videos, learning aids, games, and technology are integrated to cater to learning-style preferences of individual students. Therefore, this distance learning course is more than a workbook; it is a complete learning package.

Structure of the Learning Package

Basic Design

This learning package involves many other components in addition to the Learning Facilitator's Manual. A survey of the components will show you that the course is divided into clumps of learning called modules. For each module there are two print components: a Student Module Booklet and an Assignment Booklet.

Student Module Booklets



Student Module Booklets contain guided activities that instruct students in a relevant, realistic setting.

These booklets have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, and using advance, intermediate, and post organizers. Many other techniques are used to cater to individual learning styles and preferences. The materials have been designed to include a variety of pathways and options because they are intended for a broad range of use within and beyond Alberta.

Overview Evaluation

Section 1
Activity 1
Activity 2
etc.

Section 2
Activity 1
Activity 2
etc.

Module Summary

Appendix

The structure of the Student Module Booklets follows a systematic design. Each booklet begins with a detailed table of contents that shows the students all of the main steps; this page acts as an organizer for students. The Module Overview introduces the module topic or theme and includes a graphic representation to help visual learners and poor readers. This introduction also includes an evaluation statement, so students are informed of the weightings of each assignment.

The body of the Student Module Booklet is made up of two or more closely related sections. Each section contains student activities that develop skills and knowledge centred around a theme. The activities may include print, audio, video, computer, or Internet involvement. Computer and Internet activities are optional. At times, the student and the learning facilitator are allowed to choose the activity that best suits the student's needs and interests. Each section also includes other activities such as the Extra Help and Enrichment as optional pathways. This flexibility caters to each student's personal situation.

Following the last section is a modular summary that focuses on the skills and strategies that the student has learned. The Student Module Booklet ends with an Appendix that includes a Glossary and Suggested Answers for the self-assessment work.

Assignment Booklet



Accompanying each Student Module Booklet is an Assignment Booklet. The activities in the Assignment Booklets can be used for both formative and summative assessments. Students should complete these assignments when they have thoroughly reviewed the other module materials. You may decide to have students submit their work as soon as they have completed each sectional assignment or you may choose to let students complete all the sectional assignments within the module before evaluating their work. The Assignment Booklets have been designed for both in-

classroom use and for distance learning. If the Assignment Booklets are not being mailed, remove the green outside mailing covers before distributing the booklets to students.

Media







INTERNET

COMPUTER DISK

VIDEOCASSETTE

Internet references are found throughout the course; exploring those references, however, is optional for students. In most cases, students are given suggested topics that they may explore through the Internet's search engines. In rare cases, students might be given a specific Uniform Resource Location (URL), but they should be aware that these addresses are subject to change.

Updated information about the Learning Technologies Branch and this course and others can be found by starting at the Learning Technologies Branch's home page at http://ednet.edc.gov.ab.ca/ltb/ob/courses.html.

There are also activities in this course that require students to view videos. In some cases, the use of these videos is mandatory. These mandatory videos are listed on the following pages. It is important that you acquire these videos so students have access to them when needed. Optional videos are also mentioned at various points in the course. A list of the optional videos is also included on the following pages. More information about the videos appears later in this manual.

Materials, Media, and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
• VCR	Mandatory Video List: • Math Moves	• LFM for Mathematics 9
scientific calculator	– Equations—Solving with More than One Step• ADLC	• one complete set of Student Module Booklets (6) and Assignment Booklets (6) for
mathematical instrument set	– Joe Sceptic – Tilly Tile – Freddie Factor	each student There is a Final Test.

Videocassettes or laser videodiscs used in the course may be available from the Learning Resources Distributing Centre or ACCESS. You may also wish to call your regional library service for more information.

Optional Components

Equipment (Hardware)	Media	Materials
• VCR	Optional Video List: Multiplying Polynomials, ADLC	assorted materials
Computer	 Optional Computer Software List: a browser (such as NetscapeTM) to access the Internet a spreadsheet program (such as ClarisWorksTM) Understanding Algebra (Neufeld and Associates) Understanding Equations (Neufeld and Associates) 	
	 Understanding Exponents (Neufeld and Associates) Understanding Geometry (Neufeld and Associates) 	

The software programs *Understanding Algebra*, *Understanding Equations*, *Understanding Exponents*, and *Understanding Geometry* (Neufeld and Associates) are available from the Learning Resources Distributing Centre.

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students, so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each module, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open Learning Classroom

Open learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual's needs. But these past efforts often failed because of lack of time and lack of quality materials that conformed to Alberta specifications.

Owing to advanced educational technology and improved Alberta-specific learning packages, a student-centred approach is now possible. Improved technology now allows us to provide support to learners individually, regardless of their pace or location. A teacher cannot be in twenty-eight places at one time offering guidance. Media and a well-designed learning package, however, can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

The key to a successful open learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open learning system runs smoothly.

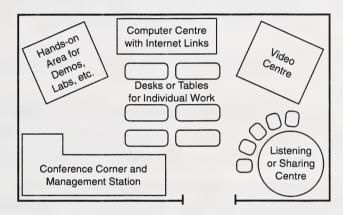
The Key to a Successful Open Learning System



Learning Package

The specially designed learning package needed for a successful open learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all of the components in the learning package available to students as needed.

If you are able to acquire media and appropriate hardware to meet your class needs, media centres can be established.



You may not have the luxury to have enough hardware to set up a permanent video or computer centre in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every three to five days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

Support

Support is definitely a key element for successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for activities included in the Student Module Booklet. Students have solutions, models, explanations, and guides included in the Appendix of every booklet. These aids are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the learning facilitator, you may need to offer more support and personal guidance to those students having difficulty. The activities include choices and pathways. If a student is struggling, you may need to encourage that student to work on all of the choices rather than on only one. This would provide additional instruction and practice in a variety of ways.

You may also have to reinforce the need for students to do the sectional activities carefully and thoroughly before attempting the assignments in the Assignment Booklet.

Another form of support is routine contact with each individual. This contact might be achieved with a biweekly conference scheduled by you; or, as students reach a certain point (e.g., after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This teaching might involve small groups or a large group. It might be used to take advantage of something timely (e.g., election, eclipse), something prescheduled like the demonstration of a process, or something involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

Management

Good management of an open learning system is essential to the success of the program. Leadership and promotion of the system are essential. The following areas also need action to ensure that the system runs smoothly:

- Scheduling, Distributing, and Managing Resources—As discussed earlier, in order to do these tasks efficiently you may need to develop media centres or a system for students to reserve the necessary resources.
- Scheduling Students—Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may want to study for long periods of time (e.g., all morning), learning facilitators should discourage this practice. Concentration, retention, and motivation are improved by taking scheduled breaks.
- Monitoring Student Progress—You will need to record the date that each student completes each module and the Final Test. Your data might also include the projected date of completion if you are using a student-contract approach.

Sample of a Student Progress Chart

Mathematic	s 9	Module 1	Module 2	Module 3	Module 4	Module 5	Module 6	Final Test
Billy Offerna	Р							
Billy Adams	Α							
Contra Domina	Р							
Louise Despins	Α							
Violet Klaissian	Р							
Violet Klaissian	Α							
P = Projected Completion Date A = Actual Completion Date								

The student could keep a personal log as well. Such tracking of data could be stored easily on a computer.

Recording Student Assessments—You will need to record the marks awarded to each student
for work completed in each module Assignment Booklet. The marks from these Assignment
Booklets will contribute to a portion of the student's final mark. Other criteria may also be
added (a special project, effort, attitude, etc.). Whatever the criteria, they should be made clear
to all students at the beginning.

Sample of a Student Assessment Chart

Mathematics 9	Module 1	Module 2	Module 3	Module 4	Module 5		Year's Average	Other	Final Test	Final Mark
Billy Adams	67	71	59	60	78	67	67			
Louise Despins	51	50	51	55	52	53	52			
Violet Klaissian	84	77	83	88	85	81	83			

Letter grading could easily be substituted.

• Recording Effectiveness of System—Keep ongoing records of how the system is working. This data will help you in future planning.

Sample of a System Assessment Chart

	Module 1							
Date	Student Module Booklet	Assignment Booklet	Resources/Media					

The Role of the Teacher in an Open Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach, it is necessary to move beyond the idea of a passive learner depending largely on a continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student centred, not teacher centred. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. In order to identify strengths and weaknesses, both students and teachers need to gather and analyse information and make decisions.

These specially designed learning packages contain many kinds of informal and formal evaluation.

Informal Evaluation

Observation

In the classroom, the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

		B. Adams	L. Despins	V. Klaissian	H. Smith	K. Dalley
1.	Does the student approach the work in a positive manner?					
2.	Is the student struggling with the reading level?					
3.	Does the student make good use of time?					
4.	Does the student apply an appropriate study method?					
5.	Can the student use references effectively?					

Observation may suggest a need for an individual interview with a student.

Individual Conferences

Individual conferences may be paced (scheduled) by the calendar, designated at certain points in the module, or they may be set up only as needed or requested.

During these conferences, the teacher can determine the student's progress and can assess the student's attitudes towards the subject, program, school, and self, as well as the student's relationship with other students. With guided questions, the teacher can encourage oral self-assessment; the student can discuss personal strengths or weaknesses in regard to the particular section, module, or subject area.

Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. Through activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude towards these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming autonomous learners. By instructing, motivating, providing positive reinforcement, and systematically supervising, learning facilitators will help students develop a positive attitude towards their own progress.

For variation, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, to have a peer assess other activities, and to become directly involved in assessing the remainder of the activities.

When the modular activities have been assessed, students should be directed to make corrections. This task should be made clear to students right from the beginning. It is important to establish the correct association between the question and the response to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for students. If students are having difficulty, more practice may be warranted, and students may need to be encouraged to select more of the choices.

Each section within a Student Module Booklet includes additional types of activities called Extra Help and Enrichment. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize the material to help you remember it?
- What steps do you follow when you complete an assignment?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this module?

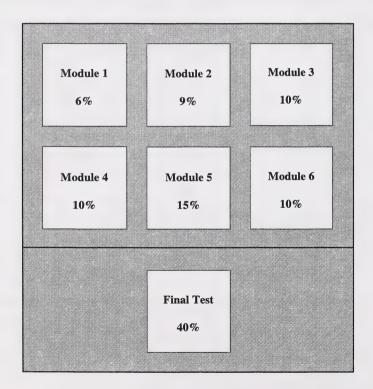
A chart or checklist could be used for recording responses.

Assignments

Each module contains a separate booklet called the Assignment Booklet. This booklet is an informal evaluation tool for assessing the knowledge or skills that the student has gained from the module. The student's mark for the module may be based solely on the outcome of learning evident in the Assignment Booklet; however, you may decide to establish a value for other variables such as attitude or effort. It is important that you establish at the beginning of the course and each module those outcomes that will be evaluated, and ensure that all students clearly understand what is expected.

Formal Evaluation

This LFM includes a formal Final Test that can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the Student Module Booklets, gives the teacher precise information concerning what each student can or cannot do. Answers, explanations, and marking guides are also included. The value of the Final Test and each module is the decision of the classroom teacher. Following is a suggestion only.



Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be repeated with the class. After the materials have been explored, a discussion might include the advantages and the disadvantages of learning independently or in small groups. The roles of the student and teacher should be analysed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude towards learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. Those skills are as follows:

- understanding and using instructional materials (table of contents, index, list of illustrations, appendices, bibliography, and glossary)
- interpreting graphs and charts
- · using reference materials
- · recognizing special symbols
- using a scientific calculator
- understanding and using different forms of media

Other general skills follow: using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total-class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.

Reading Level

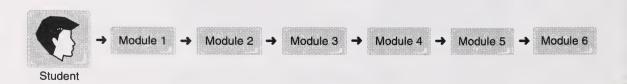
These course materials are largely print based, but poorer readers need not be discouraged. It is important that you assure students that these materials have been designed for easy reading. The authors have employed special strategies to reduce and control the reading level. Some of them are as follows:

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks

- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post organizers) to help give students a structure for incorporating new concepts
- the recognition that vocabulary and concepts are basic to understanding the content and, thus, must be handled systematically (defined in context, signalled in marginal notes or footnotes, and cited in a glossary)
- the acknowledgement that background knowledge and experience play a vital role in comprehension
- the systematic inclusion of illustrations, videos, and software to help poorer readers and visual learners
- a use of a variety of formats (paragraphs, lists, charts, etc.) to help struggling readers who do not absorb or retain main ideas easily in paragraph format
- · the inclusion of media and activity choices to encourage an active rather than passive approach
- the development of instruction in a meaningful setting rather than in a contrived, workbook style
- the use of purposeful reading, viewing, and doing to produce better interpretation of the course materials
- the recognition that students need structured learning experiences when reading, viewing, or listening to instructional materials; and the acknowledgement that the recognized reading process provides such a structured framework by emphasizing the following phases: developing pupil readiness, determining the purpose, providing guided instruction and feedback, ensuring opportunity to reread or review if necessary, and including enrichment or extensions

To help make the learning package more readable, you can begin your course or module preparation by reading (viewing, listening to) all the related materials that are going to be used. You will need this solid understanding in order to link the content to the students' experiential base. This linking may be done through class brainstorming sessions concerning the topic or by using visuals and guided questions to predict what the course or modules might be about. Such a strategy helps poorer readers strengthen their abilities to be able to predict new vocabulary.

It is recommended that you have students begin with Module 1, because this module includes basic introductory information, and it is also recommended that you end with Module 6, because this module acts as a summary or culmination.



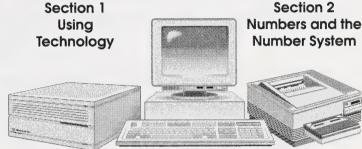
Module 1: Operations and Number Concepts

Overview

This module begins by reviewing single-step and multiple-step operations with a calculator. Students are urged to use estimation in combination with the calculator. The concept of principal square roots is introduced, and students are shown where to use principal square roots and where both positive and negative roots are applicable. The last part of the section emphasizes calculator methods and calculator efficiency. Emphasis throughout the section is on problem solving.

The second section introduces students to the different groups of numbers. Students recognize and identify numbers from each of the groups and discover how the various groups of numbers are related within the number system. Students also discover that there are other groups of numbers beyond those which they have observed thus far in their study of mathematics.

Module 1
Operations and
Number Concepts



Evaluation

The evaluation of this module will be based on three assignments:

Section 1 Assignment 40 marks
Section 2 Assignment 20 marks
Final Module Assignment 40 marks

TOTAL 100 marks

Section 1: Using Technology

Key Concepts

- · single and multiple-step calculator operations
- estimation
- · calculator methods
- · calculator efficiency
- · problem solving

The basic goals of this section are to ensure that students are able to

- · perform calculator operations with single and multiple-step expressions and problems
- · use estimation so as to recognize if calculator answers are reasonable
- · recognize and perform various calculator methods with a given expression
- · use calculators effectively and efficiently
- · use calculators in various problem solving situations

Section 1: Assignment Answer Key (40 marks)

- 1. a. (2) (5) (5) (4) = (1 mark)
 - b. $\binom{6}{2}$ $\binom{\bullet}{2}$ $\binom{\bullet}{2}$ $\binom{\bullet}{3}$ $\binom{\bullet}{3}$ $\binom{\bullet}{4}$ $\binom{\bullet}{1}$ $\binom{\bullet}{9}$ $\binom{\bullet}{3}$ $\binom{\bullet}{1}$ \binom
 - c. $(6) (\cdot) (2) (\cdot) (\cdot)$
- 2. The brackets indicate that you do the addition first. (1 mark)
- 3. **a.** 42.00 40.65 = 1.35

The net change in price per share was \$1.35. (1 mark)

b. $800 \times 1.35 = 1080$

The difference in the cost of 800 shares was \$1080. (1 mark)

c. Determine the cost of 1000 shares.

$$1000 \times 40.65 = 40650$$

The cost of 1000 shares would be \$40 650.

Determine the broker's fees.

$$100 + (40650 - 10000)1\% = 100 + (40650 - 10000)0.01$$
$$= 100 + (30650)0.01$$
$$= 100 + 306.50$$
$$= 406.50$$

The broker's fees would be \$406.50.

Total Cost =
$$40 650 + 406.50$$

= $41 056.50$

Therefore, the total cost of 1000 shares would be \$41 056.50. (2 marks)

4. **a.** Brittany:
$$\frac{3}{8} \times 6 = 3 \div 8 \times 6$$
 Taylor: $\frac{2}{5} \times 5 = 2 \div 5 \times 5$ = 2.00

$$2.25 - 2.00 = 0.25$$

Brittany spent 25¢ more than Taylor. (2 marks)

b.
$$5-2=3$$
 $3 \times \frac{1}{2} = 3 \div 2$
= 1.5

The card cost \$1.50. (1 mark)

5. **a.**
$$90^2 = 8100$$

 $100^2 = 10000$

8464 is between 8100 and 10 000; therefore, $\sqrt{8464}$ is between 90 and 100. 92 and 98 are the only numbers between 90 and 100 that will end in a 4 when multiplied by themselves. (2 marks)

b.
$$\sqrt{8464} = 92$$

Since 8464 is closer to 8100 than 10 000, choose the smaller value. (1 mark)

- 6. The negative sign in front of the radical symbol indicates that the negative of the square root is required. Therefore, the correct answer is -13. (2 marks)
- 7. Area of plot = 0.75×10000 = 7500 m^2

Length of each side =
$$\sqrt{7500}$$

= 86.602 540 38

Each side of the plot is approximately 86.6 m long. (2 marks)

8. **a.**
$$23.9 \div 5.1 \times 3.7 \doteq 25 \div 5 \times 4$$

 $\doteq 5 \times 4$
 $\doteq 20$ (2 marks)

b.
$$\frac{2.6(8.1-12.9)}{-1.8} \doteq \frac{3(8-13)}{-2}$$
$$\doteq \frac{3(-5)}{-2}$$
$$\doteq \frac{-15}{-2}$$
$$\doteq 8 \quad (2 \text{ marks})$$

9.
$$2 \times 12.99 + 3 \times 8.99 \doteq 2 \times 13 + 3 \times 9$$

 $\doteq 26 + 27$
 $\doteq 53$

The cost before taxes is approximately \$53. (2 marks)

10. a. There are two ways in which to write out the keystrokes.

Method 1



Method 2



(2 marks)

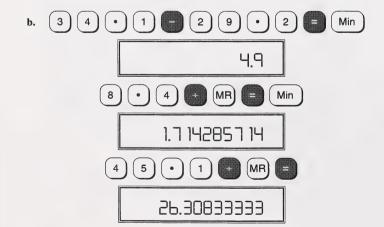
b. Methods may vary. A sample method is given.



This method uses one less keystroke. (2 marks)



The answer is 11.45, rounded to two decimal places. (2 marks)



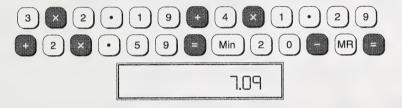
The answer is 26.31, rounded to two decimal places. (2 marks)



b. This method is just as efficient because it uses the same number of keystrokes. (1 mark)



- b. This method is just as efficient as the other two methods because it uses the same number of keystrokes. (1 mark)
- 14. Keystroke sequences may vary, but the answer should still be the same.



Maryanne will receive \$7.09 in change from a twenty. (2 marks)

Section 2: Numbers and the Number System

Key Concepts

- · number lines
- · number recognition

- · number relationships
- · group relationships

The basic goals of this section are to ensure that students

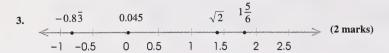
- · are able to recognize to which group a number belongs
- are able to recognize the various groups of numbers within the rational number system
- · are able to recognize numbers that are not rational numbers
- · understand the relationships between the various groups of numbers

Section 2: Assignment Answer Key (20 marks)

Integer	Rational		Other
$\frac{15}{5}$ $\sqrt{100}$ $-\sqrt{1}$	$ \begin{array}{c} -3.85 \\ 8.\overline{16} \\ \underline{15} \\ 5 \\ \sqrt{100} \\ -\sqrt{1} \end{array} $	4.2 $2\frac{1}{2}$ $0.123\ 123$ $-0.000\ 17$	π $-\sqrt{12}$ 4.121121112

(3 marks)

- 2. a. T (1 mark)
- b. F (1 mark)
- c. F (1 mark)
- d. T (1 mark)
- e. T (1 mark)



- 4. a. Natural numbers are a part of the whole numbers. (1 mark)
 - b. Integers are made up of the whole numbers and their opposites. (1 mark)
 - c. Rational numbers include all the other groups of numbers listed. (1 mark)
- 5. -8 is an integer. All integers are rational numbers. Since negative numbers are not part of whole numbers, -8 is not a whole number. (2 marks)
- **6.** Each pump can fill $\frac{1}{10}$, $\frac{1}{18}$, and $\frac{1}{22}$ of the dugout in one day.

Together, the three pumps can fill $\frac{1}{10} + \frac{1}{18} + \frac{1}{22}$ of the dugout in one day.

The following keystroke sequence can be used to perform the calculation.



It will take approximately 5 days to fill the dugout using all three pumps. (3 marks)

- 7. Students only need to show the final answer for 1 mark.
 - a. The answer is 2.94, rounded to two decimal places. (1 mark)
 - **b.** The answer is 3.36, rounded to two decimal places. (1 mark)

Final Module Assignment Answer Key (40 marks)

- 1. a. (N): 4, $\sqrt{49}$, 5, 1001
- I: $-\sqrt{81}$, -97, -2
- O: $\sqrt{3}$, 7.131131113...

(W): 0

Q: -2.5, $17\frac{1}{2}$

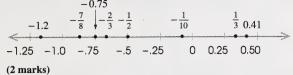
(2 marks)

- b. Stack the cups as follows:
 - cup I inside cup Q
 - cup W inside cup I
 - cup N inside cup W

Cup O will sit beside the stack of cups. (2 marks)



2.



3. Keystroke sequences may vary, but the answers should still be the same.









The answer is 289.98, rounded to two decimal places. (2 marks for method, 1 mark for final answer)

b. Method 1



Method 2



Method 3



The answer is -1.04, rounded to two decimal places. (2 marks for method, 1 mark for final answer.)

4. Raja can load $\frac{1}{2}$ of a truck in 1 h.

Julie can load $\frac{1}{4}$ of a truck in 1 h.

Carlos can load $\frac{1}{3}$ of a truck in 1 h.

Together, they can load $\frac{1}{2} + \frac{1}{4} + \frac{1}{3}$ or $1.08\overline{3}$ of a truck in 1 h.

Enter the following keystrokes on your calculator.



Together, they would load the truck in about 0.92 h. (3 marks)

5. **a.** $1:2.2228 = 1 \div 2.2228$ = 0.449 883 03

Therefore, \$1 Canadian would be equivalent to £0.4499 British. (1 mark)

b. $12.85 \times 2.2228 = 28.56298$

The shirt will cost \$28.56 Canadian. (1 mark)

- **6.** $11 \times 11 = 121$ and $(-11) \times (-11) = 121$; thus, they are both square roots of 121. (1 mark)
- 7. **a.** $-\sqrt{64} < -\sqrt{53} < -\sqrt{49}$ $-8 < -\sqrt{53} < -7$

b. $\sqrt{121} < \sqrt{130} < \sqrt{144}$ $11 < \sqrt{130} < 12$

Thus, $-\sqrt{53} \doteq -7.3$, since 53 is slightly more than one-quarter of the way between 49 and 64. (2 marks)

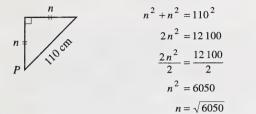
Thus, $\sqrt{130} \doteq 11.4$, since 130 is slightly more than one-third of the way between 121 and 144. (2 marks)

8. a. Keystroke sequences may vary. The following is one possible sequence of keystrokes.



- b. The wind chill is -31° C, rounded to the nearest whole degree. (1 mark)
- 9. The brackets are not required because the calculator performs the operations according to the rules of BEDMAS. (2 marks)

10. Let n represent the distance (in cm) from the corner to point P.



Point P is about 77.8 cm from the corner. (2 marks)

11. 66×66 is going to end in a 6 not a 5. The square root of 4225 must end in a 5. (2 marks)

± 77.781 745 93

- 12. A circle that fits just inside a square will have a diameter equal to the length of a side of the square and a radius equal to one-half the length of a side of the square. To find the length of a side of the square root of the area of the square.
 (2 marks)
- 13. a. Calculator sequences may vary.



Mr. Erhardt received \$54.40 for the saw. (2 marks)

b.
$$\frac{54.4}{80} \times 100 = 0.68 \times 100$$

Mr. Erhardt's selling price is 68% of the original cost price. (1 mark)

- 14. a. An advantage could be that if you make a mistake in the key entries after you have stored the intermediate value, you would only have to redo those entries after the stored entry. (2 marks)
 - b. The memory key could also be used after the keystrokes that enter the part in the first inner set of brackets in the original sequence. (The Min can be used at any point where a partial answer is displayed.) (2 marks)

Note: There are other places in the keystroke sequence where the memory key could be used.

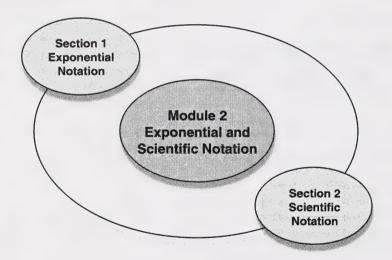
Module 2: Exponential and Scientific Notation

Overview

In this module students discover the patterns and rules involved with exponential and scientific notation. Section 1 begins with the basics of powers, some of which may have been taught in previous mathematics courses. It establishes the general pattern of exponents and shows how zero and negative exponents fit the pattern. Students are given the opportunity to evaluate powers with rational bases and integral exponents using both paper-and-pencil and calculator methods. In Activity 2, students are lead to discover the rules for multiplying and dividing exponents. In Activity 3, students carry on to discover the rules for power of a product, power of a quotient, and power of a power. The various rules are also extended to powers with non-numerical bases. Activity 4 extends the rules for order of operations (studied in Module 1) to powers and provides students with an opportunity to use their scientific calculators with order of operations. Activity 5 concludes this section with an introduction to simplifying expressions.

In Section 2, students use their knowledge of powers to help them write numbers in scientific notation. Scientific notation is then used to multiply and divide very small or very large numbers. Students are also required to develop a proficiency in the use of a scientific calculator with multiplying and dividing numbers in scientific notation.

The work with powers is very important because students will be applying this knowledge in Module 4, where they will be studying polynomials.



Evaluation

The evaluation of this module will be based on three assignments:

Section 2 Assignment	25 marks
Final Module Assignment	40 marks
TOTAL	100 marks

Section 1: Exponential Notation

Key Concepts

- · patterns in powers
- · using zero and negative integers as exponents
- · evaluating powers
- · understanding and applying the laws of exponents
- using a scientific calculator effectively with powers

The basic goals of this section are to ensure that students

- understand the meaning of powers
- are able to apply the laws of exponents to powers with rational bases and integer exponents
- · are able to use a scientific calculator effectively with powers

Section 1: Assignment Answer Key (35 marks)

1. Students may use a calculator and show only the final answer.

a.
$$21^2 = 21 \times 21$$

= 441 (1 mark)

b.
$$\left(\frac{7}{8}\right)^3 = \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8}$$

= $\frac{343}{512}$ or 0.669 921 875 (1 mark)

c.
$$(-1.7)^3 = (-1.7) \times (-1.7) \times (-1.7)$$

= -4.913 (1 mark)

d.
$$(-5)^4 = (-5) \times (-5) \times (-5) \times (-5)$$

= 625 (1 mark)

e.
$$2^{-5} = \left(\frac{1}{2}\right)^5$$

= $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
= $\frac{1}{32}$ or 0.031 25 (1 mark)

f.
$$8^0 = 1$$
 (1 mark)

2. a.
$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$
 (1 mark)

b.
$$(-8) \times (-8) \times (-8) \times (-8) = (-8)^4$$
 (1 mark)

3. a.
$$9^{.5} = 59\,049 \, (1 \, \text{mark})$$

b.
$$(-5)^{(6)} = 15625$$
 (1 mark)

4.
$$36^3 = (6^2)^3$$

Therefore, 36^3 is greater than 6^5 . (2 marks)

5. **a.**
$$(-3)^4 \times (-3)^5 = (-3)^{4+5}$$

= $(-3)^9$ (1 mark)

b.
$$4^5 \times 2^3 = (2^2)^5 \times 2^3$$

 $= 2^{10} \times 2^3$
 $= 2^{10+3}$
 $= 2^{13}$ (1 mark)

c.
$$\left(-\frac{3}{4}\right)^7 \div \left(-\frac{3}{4}\right)^{-2} = \left(-\frac{3}{4}\right)^{7-(-2)}$$

= $\left(-\frac{3}{4}\right)^{7+2}$
= $\left(-\frac{3}{4}\right)^9$ (1 mark)

d.
$$(3^2)^5 = 3^{2 \times 5}$$

= 3^{10} (1 mark)

e.
$$[(-2)^3]^4 = (-2)^{3\times4}$$

= $(-2)^{12}$ (1 mark)

f.
$$\frac{\left(4^{2}\right)^{3} \times 4^{5}}{4^{7}} = \frac{4^{6} \times 4^{5}}{4^{7}}$$
$$= 4^{6+5-7}$$
$$= 4^{11-7}$$
$$= 4^{4} \quad (1 \text{ mark})$$

- 6. a. 2 4 4 x^y 2 × 3 = (2 marks)
 - b. 8 (3 x^y 4 (4 +/_ 3)) = (2 marks
- 7. Students may have a final answer only if a calculator is used.

a.
$$7^3 - 7^2 \times 2 = 343 - 49 \times 2$$

= $343 - 98$
= 245 (1 mark)

b.
$$(5-1)^2 - 3 = 4^2 - 3$$

= 16 - 3
= 13 (1 mark)

c.
$$\left(\frac{4^2+2}{2}\right)^2 = \left(\frac{16+2}{2}\right)^2$$
$$= \left(\frac{18}{2}\right)^2$$
$$= 9^2$$
$$= 81 \quad (1 \text{ mark})$$

Standard Form	Power Form	Pattern
1000	10 ³	÷10
100	10 ²	÷10
10	10 ¹	÷10
1	10 ⁰	÷10
$\frac{1}{10}$ or 0.1	10 ⁻¹	2
$\frac{1}{100}$ or 0.01	10 ⁻²	÷10

Using patterns, it is clear that $10^0 = 1$. (2 marks)

The cycles repeat in groups of four numbers (7, 9, 3, 1). To find the last digit of 27^{18} , you must go through four complete cycles and two steps into the fifth cycle. Therefore, 27^{18} ends in 9. (2 marks)

10. a.
$$12 \times 12 = 12^2$$

= 144

b.
$$12 \times 12 \times 12 = 12^3$$

= 1728

There are 144 items in a gross. (1 mark)

There are 1728 items in a great gross. (1 mark)

11. a. The rule for dividing powers states that the bases must be the same in order to subtract the exponents. (1 mark)

b.
$$8^5 \div 4^3 = (2^3)^5 \div (2^2)^3$$

= $2^{15} \div 2^6$
= 2^{15-6}
= 2^9 (1 mark)

- 12. a. One pattern that occurs is that the two bases in each row add up to the answer. For example, $5^2 4^2 = 9$ and 5 + 4 = 9. Another pattern is that the answers appear in successive odd numbers 1, 3, 5, 7, 9,... (1 mark)
 - b. Given the pattern in question 12.a., simply add the two bases.

$$729^2 - 728^2 = 1457$$
 (1 mark)

Section 2: Scientific Notation

Key Concepts

- · writing numbers in scientific notation
- multiplying and dividing numbers written in scientific notation using a scientific calculator

The basic goals of this section are to ensure that students

- · understand what scientific notation means
- · write numbers in scientific notation and do calculations involving scientific notation using a scientific calculator

Section 2: Assignment Answer Key (25 marks)

1. a.
$$407\,000\,080 = \left(4 \times 10^{8}\right) + \left(7 \times 10^{6}\right) + \left(8 \times 10^{1}\right)$$
 (1 mark)

b. 9000.008 012 =
$$(9 \times 10^3) + (8 \times 10^{-3}) + (1 \times 10^{-5}) + (2 \times 10^{-6})$$
 (1 mark)

2. a.
$$248\,000\,000 = 2.48 \times 10^8$$
 (1 mark)

b.
$$0.000\ 000\ 099\ 2 = 9.92 \times 10^{-8}$$
 (1 mark)

c.
$$76\ 000\ 000\ 000\ 000 = 7.6 \times 10^{13}$$
 (1 mark)

3. a.
$$(4.5 \times 10^8) \times (1.2 \times 10^{-6}) = (4.5 \times 1.2) \times 10^{8 + (-6)}$$

= 5.4×10^2 (2 marks)

b.
$$\frac{\left(1.6 \times 10^{-8}\right) \times \left(4.5 \times 10^{12}\right)}{1.8 \times 10^{-3}} = \frac{1.6 \times 4.5}{1.8} \times 10^{-8+12-(-3)}$$
$$= 4 \times 10^{-8+12+3}$$
$$= 4 \times 10^{7} \quad (2 \text{ marks})$$



$$40\ 000\ 000 = 4 \times 10^{7}$$
 (2 marks)

$$0.4 = 4 \times 10^{-1}$$
 (2 marks)

5. **a.**
$$1.8 \times 10^8 = 180\ 000\ 000$$
 (1 mark)

b.
$$1.4 \times 10^{-9} = 0.000\ 000\ 001\ 4$$
 (1 mark)

6. The displayed answer represents the number 4.34×10^{10} in scientific notation and 43 400 000 000 in standard form. (2 marks)

- 7. Answers may vary depending on the calculator used.
 - a. 3 1 4 EXP 3 × 7 5 1 EXP 8 +/_ = (2 marks)
 - b. 1 6 4 EXP 1 1 5 4 1 EXP 8 = (2 marks

Note: Key symbols are not required.

8.
$$\left(4.29 \times 10^{9}\right) \div \left(2.7 \times 10^{4}\right) = 158888.\overline{8}$$

It would take the Voyager space probe 158 888. $\overline{8}\,\ h$.

Convert hours to years.

$$158 888.\overline{8} \div 24 \div 365 = 6620.\overline{370} \div 365$$
$$= 18.138 \ 001 \ 01$$
$$= 18$$

It would take the Voyager space probe a little over 18 years to reach Pluto. (2 marks)

- Answers will vary. One method for determining the height of the stack is as follows:
- Step 1: Measure the height of a can to the nearest centimetre. Divide by 100 to change the height to metres.
- Step 2: On your calculator, enter 100 billion as 1.00×10^{11} in scientific notation and multiply by the height of a can (approximately 0.12 m).
- **Step 3:** Divide by 1000 to change the answer from metres to kilometres. (2 marks)

Final Module Assignment Answer Key (40 marks)

1. **a.**
$$8^2 \times 8^3 \times 8^{-1} = 8^{2+3+(-1)}$$

= 8^4 (1 mark)

b.
$$\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{4+2+1}$$
$$= \left(\frac{1}{2}\right)^7 \quad (1 \text{ mark})$$

c.
$$(9^4)^2 \times (9^{-2})^3 = 9^{4 \times 2} \times 9^{-2 \times 3}$$

= $9^8 \times 9^{-6}$
= $9^{8+(-6)}$
= 9^2 (1 mark)

d.
$$(1.4)^6 \div (1.4)^2 = (1.4)^{6-2}$$

= $(1.4)^4$ (1 mark)

e.
$$\frac{7^{3} \times 7^{2} \times 8^{5}}{8^{4} \times 7 \times 8^{-8}} = 7^{3+2-1} \times 8^{5-4-(-8)}$$
$$= 7^{5-1} \times 8^{1+8}$$
$$= 7^{4} \times 8^{9} \quad (1 \text{ mark})$$

g.
$$\left(a^{-3}b^{5}c^{2}\right)^{2} = a^{-3\times 2}b^{5\times 2}c^{2\times 2}$$

= $a^{-6}b^{10}c^{4}$ (1 mark)

2. **a.**
$$8^2 \times 8^{-4} \times 8^5 = 8^{2+(-4)+5}$$

= 8^3
= $8 \times 8 \times 8$
= 512 (1 mark)

c.
$$2^3 + 4^3 = 8 + 64$$

= 72 (1 mark)

e.
$$9.4 \times 10^{-8} = 0.000\ 000\ 094$$
 (1 mark)

f.
$$(8 \times 10^4) + (6 \times 10^1) + (7 \times 10^{-3}) + (2 \times 10^{-4}) = 80\ 000 + 60 + 0.007 + 0.0002$$

= 80 060.0072 (1 mark)

g.
$$5^{-2} = \left(\frac{1}{5}\right)^2$$

= $\frac{1}{5} \times \frac{1}{5}$
= $\frac{1}{25}$ or 0.04 (1 mark)

3. **a.**
$$4^n \times 4^5 = 4^7$$

$$4^{n+5} = 4^7$$

What number plus five equals seven?

$$\therefore n = 2$$
 (1 mark)

f.
$$14^{-3} \div (14^{2})^{4} = 14^{-3} \div 14^{2 \times 4}$$

 $= 14^{-3} \div 14^{8}$
 $= 14^{-3-8}$
 $= 14^{-11}$ (1 mark)

h.
$$(x^3 y^2 z^{-2})^4 = x^{3\times 4} y^{2\times 4} z^{-2\times 4}$$

= $x^{12} y^8 z^{-8}$ (1 mark)

b.
$$\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^{5-3}$$
$$= \left(\frac{2}{3}\right)^2$$
$$= \frac{2 \times 2}{3 \times 3}$$
$$= \frac{4}{9} \text{ or } 0.\overline{4} \text{ (1 mark)}$$

d.
$$16^2 - 15^2 = 31$$
 (1 mark)

Simply add the two bases.

h.
$$3^4 = 3 \times 3 \times 3 \times 3$$

= 81 (1 mark)

b.
$$(4^n)^3 = 4^{-6}$$

$$4^{n \times 3} = 4^{-6}$$

What number times three equals negative six?

$$\therefore n = -2 \quad (1 \text{ mark})$$

$$\mathbf{c.} \quad \left(5^n\right)^n = 625$$

$$5^{n \times n} = 5^4$$

What number when multiplied by itself equals four?

$$\therefore n=2$$
 (1 mark)

e.
$$7^5 \times 7^n = 7^5$$

 $7^{5+n} = 7^5$

Five plus what number equals five?

$$\therefore n = 0$$
 (1 mark)

g.
$$0.000\ 000\ 014 = 1.4 \times 10^{n}$$

Count eight places from the right side of the first non-zero digit. Since the count is to the left, then the exponent of the power of 10 is negative.

$$\therefore n = -8 \quad (1 \text{ mark})$$

- **4. a.** $0.000\ 000\ 000\ 007\ 5 = 7.5 \times 10^{-12}$ (1 mark)
 - **b.** $82\,700\,000\,000 = 8.27 \times 10^{10}$ (1 mark)

$$10\ 080\ 000 = 1.008 \times 10^{7}$$
 (1 mark)

d.
$$(n)^{-2} = \left(\frac{1}{4}\right)^2$$

Since the signs of the exponents are opposite, the bases are reciprocals.

$$\therefore n = 4 \quad (1 \text{ mark})$$

f. Write $\frac{27}{8}$ as a power.

$$\frac{27}{8} = \left(\frac{3}{2}\right)^3$$

Since the base $\frac{2}{3}$ and the base $\frac{3}{2}$ are reciprocals, the exponents are additive inverses (opposite in sign) of each other.

$$\therefore n = -3 \quad (1 \text{ mark})$$

h.
$$1\,000\,000\,000 = n \times 10^9$$

1 000 000 000 has only 1 as a non-zero digit.

$$\therefore n=1$$
 (1 mark)

6. All powers of whole numbers with an exponent of 5 end with the same digit that the base does. For example, $1^5 = 1$, $2^5 = 32$, and $3^5 = 243$.

Thus, 11^5 ends in 1, 12^5 ends in 2, 13^5 ends in 3, and 14^5 ends in 4.

Since 1+2+3+4=10, then $11^5+12^5+13^5+14^5$ ends in 0. (3 marks)

- 7. $2^{10} \times 5^{10} = (2 \times 5)^{10}$ = 10^{10} = $10\ 000\ 000\ 000\ (2\ marks)$
- 8. $\left(7.38 \times 10^{19}\right) \div \left(5.7 \times 10^{15}\right) = 12\ 947.368\ 42$ = 13 000

Therefore, the moon is about 13 000 times heavier than Earth's atmosphere. (2 marks)

9. The numbers must first be written in scientific notation.

$$(2.3716 \times 10^{14}) \div (2.45 \times 10^{5})$$

Therefore, the keystrokes are as follows:

10. a.
$$\frac{1.0 \times 10^{-4}}{5.0 \times 10^{-6}} = 20$$

The diameter of the human hair is 20 times greater than the spider silk. (2 marks)

b.
$$10 \times 20 = 200$$

To represent the cross-section of a human hair, you would need a circle with a diameter of 200 mm (or 20 cm). (1 mark)

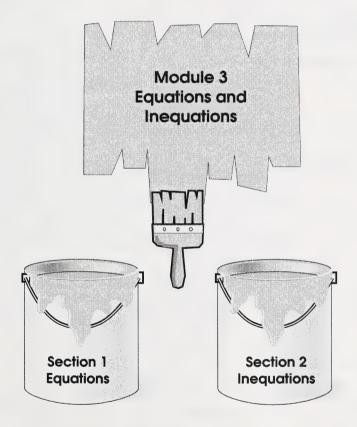
Module 3: Equations and Inequations

Overview

Section 1 begins by solving various forms of equations using models and paper and pencil. The students apply these techniques to solving numerous problems involving equations. Solving problems involving formulas is also included throughout this section.

Section 2 introduces inequalities and the terms and symbols associated with them. Students solve inequations using techniques and approaches similar to those used for solving equations. This section includes activities explaining what the solutions to inequations really mean and how they can be graphed on a number line.

The understanding of equations and inequations and the ability to express problem situations using equations and inequations are powerful and necessary tools that the students can utilize as they attack a variety of problems.



Evaluation

The evaluation of this module will be based on three assignments:

Section 1 Assignment 45 marks
Section 2 Assignment 30 marks
Final Module Assignment 25 marks

TOTAL 100 marks

Mandatory Video

Equations—Solving with More than One Step, Math Moves series, ACCESS. This video is available from the Learning Resources Distributing Centre.

Section 1: Equations

Key Concepts

- · solving equations using models
- · solving equations using paper and pencil
- solving problems through the use of equations
- · solving problems involving formulas

The basic goals of this section are to ensure that students

- · understand and make use of the language of algebra
- · are able to solve various forms of equations
- are able to use equations to solve various problems

Section 1: Assignment Answer Key (45 marks)

1. **a.**
$$3a + 5a = -72$$

 $8a = -72$
 $\frac{8a}{8} = \frac{-72}{8}$

$$\frac{8a}{8} = \frac{-72}{8}$$

$$a = -9 \quad (2 \text{ marks})$$

b.
$$3(n-4)=15$$

 $3n-12=15$
 $3n-12+12=15+12$

$$3n = 27$$

$$\frac{3n}{3} = \frac{27}{3}$$

n=9 (2 marks)

c.
$$3b+5=7b-11$$

 $3b+5-5=7b-11-5$
 $3b=7b-16$

$$3b-7b = 7b-16-7b$$

 $-4b = -16$

$$\frac{-4b}{-4} = \frac{-16}{-4}$$

$$b = 4 \quad (2 \text{ marks})$$

d.
$$\frac{5}{p} > \frac{2}{6}$$

$$5 \times 6 = 2 \times p$$

$$30 = 2 p$$
$$2 p = 30$$

$$\frac{2p}{2} = \frac{30}{2}$$

$$p = 15$$
 (2 marks)

e.
$$\frac{2}{n+3} \rightarrow \frac{4}{4n}$$

$$2 \times 4n = 4(n+3)$$

$$8n = 4n + 12$$

$$8n - 4n = 4n + 12 - 4n$$

$$4n = 12$$

$$\frac{4n}{4} = \frac{12}{4}$$

$$n=3$$
 (2 marks)

2. a.
$$2y-5=4y$$

 $2y-5+5=4y+5$

$$2y = 4y + 5$$
$$2y - 4y = 4y + 5 - 4y$$

$$-2y = 5$$

$$\frac{-2y}{-2} = \frac{5}{-2}$$

$$y = -2.5$$
 (2 marks)

b. Method 1

$$\frac{y-2}{5} = \frac{3y+4}{10}$$

$$\frac{y-2}{5} \times 10 = \frac{3y+4}{10} \times 10$$

$$2y-4 = 3y+4$$

$$2y-4+4 = 3y+4+4$$

$$2y = 3y+8$$

$$2y-3y = 3y+8-3y$$

$$-y = 8$$

$$y = -8$$

Method 2

$$\frac{y-2}{5} \longrightarrow \frac{3y+4}{10}$$

$$(y-2)\times 10 = (3y+4)\times 5$$

$$10y-20=15y+20$$

$$10y-20+20=15y+20+20$$

$$10y-15y=15y+40-15y$$

$$-5y=40$$

$$\frac{-5y}{-5} = \frac{40}{-5}$$

$$y=-8 \quad (2 \text{ marks})$$

Verification

LS	RS
$\frac{y-2}{5}$	$\frac{3y+4}{10}$
$=\frac{-8-2}{5}$	$=\frac{3(-8)+4}{10}$
$=\frac{-10}{5}$	$=\frac{-24+4}{10}$
=-2	$=\frac{-20}{10}$
	=-2
LS :	= RS
(2 marks)	

- 3. a. 3n-2=2n+5 (1 mark)
 - **b.** $n+3n=120\ 000$ or $4n=120\ 000$ (1 mark)
- 4. a. LS RS 3(x-2) 12 = 3[(-2)-2]

=3(-4)

= -12

LS ≠ RS

b. LS RS 9(4+x) 18 = 9[4+(-2)] = 9(2)= 18

This equation does not have -2 for a solution. (1 mark)

This equation has -2 for a solution. (1 mark)

LS

RS

- 5. Represent the algebraic equation with three positive cylinders on the left and five positive cylinders and two positive counters on the right. Add five negative cylinders to each side. Combine equal numbers of positive and negative cylinders and remove the pairs. Replace each side of the equation with its inverse. Divide the remaining cylinders and counters into two groups. The solution to the equation is the number of counters in one group. (4 marks)
- 6. a. $C = 2\pi r$ $\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$ $r = \frac{C}{2\pi}$ (2 marks)
- b. $V = \frac{1}{2}abh$ $V \times 2 = \frac{1}{2}abh \times \frac{1}{2}$ 2V = abh $\frac{2V}{ab} = \frac{abh}{ab}$ $h = \frac{2V}{ab} \quad (2 \text{ marks})$

7. Method 1

$$A = \frac{h(a+b)}{2}$$

$$96 = \frac{8(10+b)}{2}$$

$$96 \times 2 = \frac{80+8b}{2} \times 2$$

$$192 = 80+8b$$

$$192 - 8b = 80 + 8b - 8b$$

$$192 - 8b - 192 = 80 - 192$$

$$-8b = -112$$

$$\frac{-8b}{-8} = \frac{-112}{-8}$$

$$b = 14$$

Method 2

$$A = \frac{h(a+b)}{2}$$

$$A \times 2 = \frac{h(a+b)}{2} \times \frac{1}{2}$$

$$2A = h(a+b)$$

$$\frac{2A}{h} = \frac{\frac{1}{2}(a+b)}{\frac{1}{2}}$$

$$\frac{2A}{h} = a+b$$

$$\frac{2A}{h} - a = b$$

$$b = \frac{2A}{h} - a$$

$$= \frac{2(96)}{8} - 10$$

$$= 24 - 10$$

$$= 14$$

The length of the base is 14 cm. (3 marks)

- 8. Students must solve any three of the following problems. An algebraic solution is shown for all the problems. Students may use another method to solve one of them.
 - **a.** Let n be the number of tickets sold last year. Thus, 2n is the number of advance tickets sold this year.

$$2n + 235 = 825$$

$$2n + 235 - 235 = 825 - 235$$

$$2n = 590$$

$$\frac{2n}{2} = \frac{590}{2}$$

$$n = 295$$

There were 295 tickets sold last year. (4 marks)

b.

Candy	Number	Cost of Each Candy (¢)	Total Cost (¢)
Jawbreakers	n	5	5 n
Licorice	n+5	8	8(n+5)

The problem can be described and solved as follows:

$$5n+8(n+5) = 170$$

$$5n+8n+40 = 170$$

$$13n+40 = 170$$

$$13n+40-40 = 170-40$$

$$13n = 130$$

$$\frac{13n}{13} = \frac{130}{13}$$

$$n = 10$$

Now find the number of licorice Janelle bought.

$$n+5=10+5$$
$$=15$$

Janelle bought 10 jawbreakers and 15 licorice. (4 marks)

c.

Person	Present Age	Age in Eight Years
Joan	n	n + 8
Bret	n+9	n+17

The problem can be described and solved as follows:

$$6(n+17) = 7(n+8)$$

$$6n+102 = 7n+56$$

$$6n+102-102 = 7n+56-102$$

$$6n = 7n-46$$

$$6n-7n = 7n-46-7n$$

$$-n = -46$$

$$n = 46$$

Now find Bret's present age.

$$n+9 = 46+9$$
$$= 55$$

Joan is 46 years old, and Bret is 55 years old. (4 marks)

d. Let n be the amount of space (in metres) Mrs. Habscheid has for her garden. Thus, $\frac{n}{2}$ or $\frac{1}{2}n$ is the space for vegetables, $\frac{n}{4}$ or $\frac{1}{4}n$ is the space (in metres) for fruit, and $\frac{n}{6}$ or $\frac{1}{6}n$ is the space (in metres) for flowers.

The problem can be described and solved as follows:

$$\frac{n}{2} + \frac{n}{4} + \frac{n}{6} + 8 = n$$

$$\left(\frac{n}{2} + \frac{n}{4} + \frac{n}{6} + 8\right) \times 12 = n \times 12$$

$$\left(\frac{n}{2} \times 12\right) + \left(\frac{n}{4} \times 12\right) + \left(\frac{n}{4} \times 12\right) = 12n$$

$$6n + 3n + 2n + 96 = 12n$$

$$11n + 96 = 12n$$

$$11n + 96 = 96$$

$$11n = 12n - 96$$

$$11n - 12n = 12n - 96 - 12n$$

$$-n = -96$$

$$n = 96$$

Mrs. Habsheid has 96 m² for her garden. (4 marks)

Section 2: Inequations

Key Concepts

- · understanding and interpreting inequalities
- · solving inequations using formal procedures
- · solving problems involving inequations
- plotting solution sets of inequations on a number line

The basic goals of this section are to ensure that students

- · understand the concept of inequalities
- · are able to solve various forms of inequations
- · understand how to interpret inequalities and plot their solutions on a number line
- · are able to use inequations to solve various problems

Section 2: Assignment Answer Key (30 marks)

1. a.
$$m+7<11$$
 b. $2n \ge -12$ c. $y-3>-1$ $m+7-7<11-7$ $2n \ge -12$ $y-3+3>-1+3$ $y>2$ (1 mark) $n \ge -6$ (1 mark)

2. a.
$$-4a+8>-12$$

 $-4a+8-8>-12-8$
 $-4a>-20$
 $\frac{-4a}{-4}<\frac{-20}{-4}$
 $a<5$ (2 marks)

b.
$$4(a-3) \le -8$$

 $4a-12 \le -8$
 $4a-12+12 \le -8+12$
 $4a \le 4$
 $\frac{4a}{4} \le \frac{4}{4}$
 $a \le 1$ (2 marks)

c.
$$-12x < 54$$

 $\frac{-12x}{-12} > \frac{54}{-12}$
 $x > -4.5$ (2 marks)

3. a.
$$3(n+2) < 4n$$
 (1 mark)

b.
$$n+2n+n+(n+4)+(n+2) \le 30$$
 or $6n+6 \le 30$ (1 mark)

LS RS

$$x+2$$
 6
 $=(4)+2$
 $=6$
LS = RS

Thus, x = 4 is not a solution to the inequation x + 2 > 6. (1 mark)

c. LS RS
$$\begin{array}{c|cc}
b-1 & 3 \\
\hline
= (4)-1 & \\
= 3 & \\
\end{array}$$
LS = RS

Thus, b = 4 is not a solution to the inequation b-1>3. (1 mark)

5. a.
$$-4x \le 28$$

$$\frac{-4x}{-4} \ge \frac{28}{-4}$$

$$x \ge -7 \quad (1 \text{ mark})$$

$$-9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4$$

$$(1 \text{ mark})$$

Thus, n = 4 is not a solution to the inequation $-3n \le -15$. (1 mark)

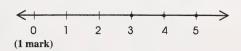
d. LS RS

$$4b-8$$
 12
 $= 4(4)-8$
 $= 16-8$
 $= 8$
LS < RS

Thus, b = 4 is a solution to the inequation 4b - 8 < 12. (1 mark)

b.
$$3x+4>10$$

 $3x+4-4>10-4$
 $3x>6$
 $\frac{3x}{3}>\frac{6}{3}$
 $x>2$ (1 mark)



- 6. a. $x \le 2$ (1 mark)
 - b. The graph points to the left and starts with a solid dot at 2, indicating that the values are less than and equal to 2. Therefore, the inequality is $x \le 2$. (2 marks)
- Students are to answer two of the following problems. An algebraic solution is shown for all problems, but students may solve one of them using some other method.
 - a. Let n be the number of children on the top floor. Thus, 4n is the number of children on the bottom floor, and n+8 is the number of children on each of the middle floors.

$$n+4n+(n+8)+(n+8)<100$$

$$7n+16<100$$

$$7n+16-16<100-16$$

$$7n<84$$

$$\frac{7n}{7}<\frac{84}{7}$$

$$n<12$$

Therefore, the old lady could have 11 children on the top floor, 11+8=19 children on each of the middle two floors, and 4(11)=44 children on the bottom floor. (4 marks)

b. Let n be the number.

$$2n+7 \ge 35$$

$$2n+7-7 \ge 35-7$$

$$2n \ge 28$$

$$\frac{2n}{2} \ge \frac{28}{2}$$

$$n \ge 14$$

The number is greater than or equal to 14. (4 marks)

c. Let n be the amount of money Troy had in his savings account.

$$n-25 > 140$$

$$n-25+25 > 140+25$$

$$n > 165$$

Troy could have had a minimum of \$165.01 in his account. (4 marks)

Final Module Assignment Answer Key (25 marks)

1. a.
$$4(y+3) = 28$$

 $4y+12 = 28$
 $4y+12-12 = 28-12$
 $4y=16$
 $\frac{4y}{4} = \frac{16}{4}$
 $y = 4$ (2 marks)

c.
$$\frac{b}{2} + \frac{b}{3} + \frac{b}{4} = 26$$

$$\left(\frac{b}{2} + \frac{b}{3} + \frac{b}{4}\right) \times 12 = 26 \times 12$$

$$\left(\frac{b}{2} \times 12\right) + \left(\frac{b}{3} \times 12\right) + \left(\frac{b}{3} \times 12\right) = 312$$

$$6b + 4b + 3b = 312$$

$$13b = 312$$

$$\frac{13b}{13} = \frac{312}{13}$$

$$b = 24$$

Thus, n = -6 makes the inequation true. (1 mark)

3. Method 1

Let *n* be the length. Thus, n-3 is the width.

$$n - 3$$

$$n+n+(n-3)+(n-3)=38$$

 $4n-6=38$

(3 marks)

b.
$$3a+7=5a-9$$

 $3a+7-7=5a-9-7$
 $3a=5a-16$
 $3a-5a=5a-16-5a$
 $-2a=-16$
 $\frac{-2a}{-2}=\frac{-16}{-2}$
 $a=8$ (2 marks)

d.
$$-4n-2 \ge 22$$

 $-4n-2+2 \ge 22+2$
 $-4n \ge 24$
 $\frac{-4n}{-4} \le \frac{24}{-4}$
 $n \le -6$ (2 marks)

b. LS RS
$$3(2m+3) 45$$

$$= 3[2(7)+3] = 3(14+3)$$

$$= 3(17) = 51$$
LS > RS

Thus, m = 7 does not make the equation true. (1 mark)

Method 2

Let n be the width. Thus, n+3 is the length.

$$n+n+(n+3)+(n+3)=38$$

 $4n+6=38$

4. a.
$$I = prt$$

$$\frac{I}{pt} = \frac{prt}{pt}$$

$$\frac{I}{pt} = r$$

$$r = \frac{I}{pt}$$
 (2 marks)

b.
$$d = \frac{v}{t}$$

$$d \times t = \frac{v}{\lambda} \times \frac{1}{\lambda}$$

$$dt = v$$

$$\frac{dt}{d} = \frac{v}{d}$$

$$t = \frac{v}{d} \quad (2 \text{ marks})$$

5. a. $n \ge 2$ (1 mark)

- **b.** n < 2, where n is an integer, or $n \le 1$, where n is an integer (1 mark)
- Person Present Age 15
 Years Ago

 Alston n n-15

 Dr. Choi 2n 2n-15

$$2n-15 = 5(n-15)$$

$$2n-15 = 5n-75$$

$$2n-15+15 = 5n-75+15$$

$$2n = 5n-60$$

$$2n-5n = 5n-60-5n$$

$$-3n = -60$$

$$\frac{-3n}{-3} = \frac{-60}{-3}$$

$$n = 20$$

Now find Dr. Choi's present age.

$$2n = 2(20)$$
$$= 40$$

Alston is presently 20 years old, and Dr. Choi is presently 40 years old. (3 marks)

b. Let n be the capacity (in mL) of the small jar. Thus, 2n+4 is the capacity (in mL) of the other jar.

$$n + (2n+4) < 710$$

$$3n+4 < 710$$

$$3n+4-4 < 710-4$$

$$3n < 706$$

$$\frac{3n}{3} < \frac{706}{3}$$

$$n < 235.\overline{3}$$

The small jar can hold a maximum of 235 mL. (3 marks)

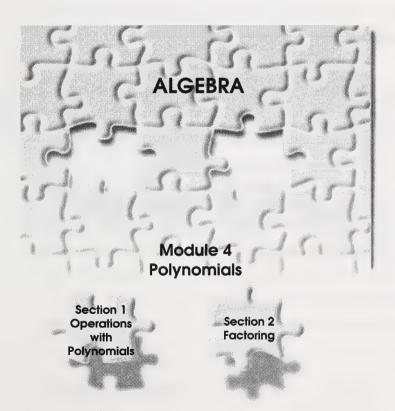
Module 4: Polynomials

Overview

This module begins by introducing some of the terminology of polynomials. The students then become familiar with how to model polynomials using algebra tiles. They analyse various ways of representing zero and discover what like terms are and how to combine them using algebra tiles. Evaluating polynomials completes the first activity. The remainder of Section 1 is devoted to performing computations with various kinds of polynomials. The students use algebra tiles to manipulate the polynomials so they can understand how computations of adding, subtracting, multiplying, and dividing with polynomials are done, thus leading to performing computations using paper and pencil.

Section 2 introduces the students to factoring. They review what factoring means with whole numbers and then apply these concepts to algebraic terms and polynomials. Factoring includes finding the common factors and factoring out the GCF of a number of terms and polynomials. Common binomial factors and some simple trinomials are also examined.

This module gives the students a good understanding of the basics of polynomial algebra. They will be able to carry these concepts forward and build upon them as they further their mathematical experiences in future mathematics courses.



Evaluation

The evaluation of this module will be based on four assignments:

Section 1 Assignment	45 marks
Section 2 Assignment	20 marks
Final Module Assignment	35 marks

100 marks TOTAL

Mandatory Videos

- Joe Sceptic, ADLC. This video is available from the Learning Resources Distributing Centre.
- Tilly Tile, ADLC. This video is available from the Learning Resources Distributing Centre.
- Freddie Factor, ADLC. This video is available from the Learning Resources Distributing Centre.

Optional Video

Multiplying Polynomials, ADLC. This video is available from the Learning Resources Distributing Centre.

Section 1: Operations with Polynomials

Key Concepts

- · the terminology of polynomials
- · evaluating polynomials
- · multiplying binomials
- · dividing polynomials by monomials · using algebra tiles to justify the algebraic process for computation with polynomials
- · modelling polynomials
- · adding and subtracting polynomials
- · multiplying monomials and multiplying polynomials by monomials

The basic goals of this section are to ensure that students

- · understand and are able to use the basic vocabulary of polynomials
- · are able to model various polynomials and use the models to manipulate polynomials
- · are able to evaluate polynomials
- are able to add and subtract polynomials of any type
- · are able to multiply binomials
- · are able to multiply any type of polynomial by a monomial
- · are able to divide a polynomial by a monomial
- are able to explain how algebra tiles can be used to justify the algebraic process for computation with polynomials

Section 1: Assignment Answer Key (45 marks)

Answers may vary for each question except for question 1.a.

a.
$$3x^2y$$
 $\left(\frac{1}{2} \text{ mark}\right)$

b.
$$4+2a+3a^3 \left(\frac{1}{2} \text{ mark}\right)$$

c.
$$4a^2b+7(\frac{1}{2} \text{ mark})$$

d.
$$-8x^2yz$$
 $\left(\frac{1}{2} \text{ mark}\right)$

2. a.
$$3x^2 + 2x - 4$$
 (1 mark)

b.
$$2x^2 - x + 2$$
 (1 mark)

3. **a.**
$$2c^2 + 7 - 3b + c^2 + b = 2c^2 + c^2 - 3b + b + 7$$

= $3c^2 - 2b + 7$ (1 mark)

Substitute b = -1 and c = 3.

$$3c^{2}-2b+7=3(3)^{2}-2(-1)+7$$

= $3(9)+2+7$
= $27+2+7$
= 36 (1 mark)

b.
$$a^2b-b^2+3c-b^2+c-a^2b+1=a^2b-a^2b-b^2-b^2+3c+c+1$$

= $-2b^2+4c+1$ (1 mark)

Substitute b = -1 and c = 3.

$$-2b^{2} + 4c + 1 = -2(-1)^{2} + 4(3) + 1$$
$$= -2(1) + 12 + 1$$
$$= -2 + 12 + 1$$
$$= 11 \quad (1 \text{ mark})$$

4.

(1 mark)

- 5. a. One $-x^2$ -tile, two x-tiles, and one 1-tile need to be added. (1 mark)
 - **b.** One -x-tile and one 1-tile need to be added. (1 mark)

6. a.
$$(2x^2 + 7x + 6) + (3x^2 - 5x + 1) = 2x^2 + 7x + 6 + 3x^2 - 5x + 1$$

= $2x^2 + 3x^2 + 7x - 5x + 6 + 1$
= $5x^2 + 2x + 7$ (1 mark)

b.
$$(4y^2 - 7y + 5) + (5y - 3) = 4y^2 - 7y + 5 + 5y - 3$$

= $4y^2 - 7y + 5y + 5 - 3$
= $4y^2 - 2y + 2$ (1 mark)

c.
$$3x^2y + 5xy - 2y^2 + 8$$

 $+ (-2x^2y - xy + 5y^2 + 1)$
 $-x^2y + 4xy + 3y^2 + 9$ (1 mark)

d.
$$(3x^3 + 4x^2 - 8) + (2x^2 - 5x + 1) = 3x^3 + 4x^2 - 8 + 2x^2 - 5x + 1$$

= $3x^3 + 4x^2 + 2x^2 - 5x - 8 + 1$
= $3x^3 + 6x^2 - 5x - 7$ (1 mark)

7. **a.**
$$(3x^2 - x + 4) - (2x^2 + 4x - 3) = 3x^2 - x + 4 - 2x^2 - 4x + 3$$

= $3x^2 - 2x^2 - x - 4x + 4 + 3$
= $x^2 - 5x + 7$ (1 mark)

b.
$$(-4y^2 + 4y - 3) - (2y^2 + 8) = -4y^2 + 4y - 3 - 2y^2 - 8$$

= $-4y^2 - 2y^2 + 4y - 3 - 8$
= $-6y^2 + 4y - 11$ (1 mark)

c.
$$2a^{2}b+7ab-4b+3$$

 $-(a^{2}b-2ab+5b-8)$ $2a^{2}b+7ab-4b+3$
 $+(-a^{2}b+2ab-5b+8)$
 $a^{2}b+9ab-9b+11$ (1 mark)

d.
$$(4ab-b+8)-(4ab+b-8) = 4ab-b+8-4ab-b+8$$

= $4ab-4ab-b+8+8$
= $-2b+16$ (1 mark)

8. **a.**
$$2m^2 \times 3m = (2)(3)(m^2)(m)$$

= $6m^3$ (1 mark)

b.
$$(4ab^2c)(-3abc^3) = (4)(-3)(a)(a)(b^2)(b)(c)(c^3)$$

= $-12a^2b^3c^4$ (1 mark)

c.
$$(-2y(y^2-2y+3)=(-2y)(y^2)+(-2y)(-2y)+(-2y)(3)$$

= $(-2y)^3+4y^2-6y$ (1 mark)

d.
$$(4y)(-3xy)(2x^2y) = (4)(-3)(2)(x)(x^2)(y)(y)(y)$$

= $-24x^3y^3$ (1 mark)

e.
$$(x+5)(x+3) = (x)(x)+(x)(3)+(5)(x)+(5)(3)$$

= $x^2 + 3x + 5x + 15$
= $x^2 + 8x + 15$ (1 mark)

f.
$$(2x+3)(x-1) = (2x)(x)+(2x)(-1)+(3)(x)+(3)(-1)$$

= $2x^2-2x+3x-3$
= $2x^2+x-3$ (1 mark)

g.
$$3(x+4)+2(x^2-6x-3)=(3)(x)+(3)(4)+2(x^2)+(2)(-6x)+(2)(-3)$$

$$=3x+12+2x^2-12x-6$$

$$=2x^2+3x-12x+12-6$$

$$=2x^2-9x+6 (1 mark)$$

h.
$$(x-8)(x+8) = (x)(x)+(x)(8)+(-8)(x)+(-8)(8)$$

= $x^2 + 8x - 8x - 64$
= $x^2 - 64$ (1 mark)

9. **a.**
$$\frac{10x^2 + 15x}{5x} = \frac{10x^2}{5x} + \frac{15x}{5x}$$
$$= 2x + 3$$
 (1 mark)

b.
$$(18x^4 + 6x^3 - 9x^2 + 12x) \div 3x = \frac{18x^4}{3x} + \frac{6x^3}{3x} + \frac{-9x^2}{3x} + \frac{12x}{3x}$$

= $6x^3 + 2x^2 - 3x + 4$ (1 mark)

c.
$$\frac{24 m^3 - 16 m^2 + 32 m}{8 m} = \frac{24 m^3}{8 m} + \frac{-16 m^2}{8 m} + \frac{32 m}{8 m}$$
$$= 3 m^2 - 2 m + 4 \quad (1 \text{ mark})$$

10. a.
$$(8x^2 - 12x) \div 2x = \frac{8x^2}{2x} + \frac{-12x}{2x}$$

= $4x - 6$ (1 mark)

b.
$$(6x-9) \div 3 = \frac{6x}{3} + \frac{-9}{3}$$

= 2x-3 (1 mark)

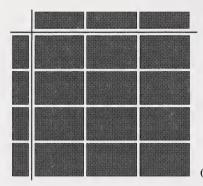
11. a.
$$(2x+3)(2x+3) = (2x)(2x)+(2x)(3)+(3)(2x)+(3)(3)$$

= $4x^2+6x+6x+9$
= $4x^2+12x+9$ square units (1 mark)

b.
$$(2x)(2x)+(2x)(4x) = 4x^2 + 8x^2$$

= $12x^2$ square units (1 mark)

12. a.



(1 mark)

- b. A new shape was chosen to represent y because it is a different variable. (1 mark)
- c. The dimensions of one product shape is xy. Since there are twelve product areas, the product of 3x and 4y is 12xy. (1 mark)

13. a.
$$\ell = \frac{9a^2 + 6a}{3a}$$

$$= \frac{9a^2}{3a} + \frac{6a}{3a}$$

$$= 3a + 2 \text{ units } (1 \text{ mark})$$

b.
$$P = 2w + 2\ell$$

= $2(3a) + 2(3a + 2)$
= $6a + 6a + 4$
= $12a + 4$ units (1 mark)

14. a.
$$\left(8x^2y + 4xy^2 - 12x^2y^2\right) + 4xy = \frac{8x^2y}{4xy} + \frac{4xy^2}{4xy} + \frac{-12x^2y^2}{4xy}$$

= $2x + y - 3xy$ (1 mark)

b. Method 1

Multiply 4xy by 2x + y - 3xy to see if you get the original polynomial.

$$4xy(2x+y-3xy) = 4xy(2x)+4xy(y)+4xy(-3xy)$$
$$= 8x^2y+4xy^2-12x^2y^2$$

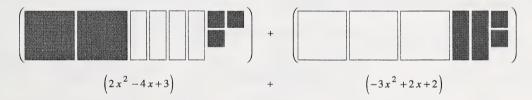
Method 2

Substitute a number for x and a number for y; then evaluate to see if the left side equals the right side. Try x = 2 and y = 3.

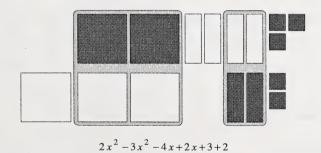
LS	RS
$8x^2y + 4xy^2 - 12x^2y^2$	2x + y - 3xy
4 xy	=2(2)+3-3(2)(3)
$=\frac{8(2)^2(3)+4(2)(3)^2-12(2)^2(3)^2}{12(2)^2(3)^2}$	= 4+3-18
= 4(2)(3)	. = -11
$=\frac{8(4)(3)+4(2)(9)-12(4)(9)}{4(2)(3)}$	
$=\frac{96+72-432}{24}$	
$=\frac{-264}{24}$	
=-11	
LS :	= RS

(2 marks)

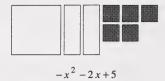
15. Step 1: Redraw the model using the opposite of each term in the second polynomial, and change the operation to addition.



Step 2: Use the zero principle to collect the like tiles (terms) together.



Step 3: Combine like terms by eliminating the zero pairs and drawing the remaining tiles.



(3 marks)

Section 2: Factoring

Key Concepts

- · understanding what factors are and what factoring involves
- · factoring by finding a common factor

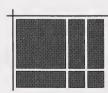
- · using algebra tiles to justify the algebraic process of factoring
- · factoring by grouping

The basic goals of this section are to ensure that students

- · understand what factors are and what it means to factor a polynomial
- · are able to find the GCF
- · are able to factor polynomials by finding a common factor
- are able to factor trinomials of the form $x^2 + bx + c$

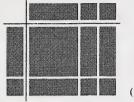
Section 2: Assignment Answer Key (20 marks)

- When asked to factor $6x^2 + 12x$ completely, it means you must find the GCF of $6x^2$ and 12x first; then write out the expression as a product of two factors. (2 marks)
- a. $2x^2y$ (1 mark)
- **b.** $5ab^2c$ (1 mark) **c.** $6x^2y$ (1 mark)
- **a.** $20a^2b^2 + 15ab^3 10a^2b^3 = 5ab^2$ (4a + 3b 2ab) (1 mark) **b.** $12b^2 32b = 4b$ (3b-8) (1 mark)
- **Step 1:** Lay out the tiles required to illustrate $x^2 + 3x + 2$ in the shape of a rectangle.



(1 mark)

Step 2: Draw the tiles representing the length and the width.



(1 mark)

Step 3: State the factors from the tiles representing the dimensions.

The factors are x+2 and x+1. (1 mark)

5. **a.**
$$7x^2 + 14x - 21 = 7(x^2 + 2x - 3)$$

= $7(x+3)(x-1)$ (2 marks)

b.
$$10x^2y + 8xy^2 + 6xy = 2xy(5x + 4y + 3)$$
 (2 marks)

c.
$$x(x+3)+5(x+3)=(x+3)(x+5)$$
 (2 marks)

d.
$$(3a^2 + 6a) + (5a + 10) = 3a(a + 2) + 5(a + 2)$$

= $(a + 2)(3a + 5)$ (2 marks)

6. **a.**
$$\frac{12ab^2}{8a^2b} = \frac{\frac{1}{4ab}(3b)}{\frac{2ab}{1}(2a)}$$
$$= \frac{3b}{2a} \text{ (1 mark)}$$

b.
$$\frac{12mn}{18m^2n^3} = \frac{\frac{1}{6mn}(2)}{6mn(3mn^2)}$$
$$= \frac{2}{3mn^2} (1 \text{ mark})$$

Final Module Assignment Answer Key (35 marks)

- 1. a. a second-degree trinomial in one variable (written in descending order) (1 mark)
 - b. a fifth-degree binomial in two variables (written in ascending order) (1 mark)

2. **a.**
$$4mn - 6m + 2n^2 + 3m - 2 = 4mn - 6m + 3m + 2n^2 - 2$$

= $4mn - 3m + 2n^2 - 2$ (1 mark)

Substitute m = -2 and n = 3.

$$4mn-3m+2n^{2}-2=4(-2)(3)-3(-2)+2(3)^{2}-2$$

$$=4(-2)(3)-3(-2)+2(9)-2$$

$$=-24+6+18-2$$

$$=-2 (1 mark)$$

b.
$$18-2m^3+m^2+2m-12+5m=18-12-2m^3+m^2+2m+5m$$

= $6-2m^3+m^2+7m$ (1 mark)

Substitute m = -2.

$$6-2m^{3}+m^{2}+7m=6-2(-2)^{3}+(-2)^{2}+7(-2)$$

$$=6-2(-8)+4+7(-2)$$

$$=6+16+4-14$$

$$=12 \quad (1 \text{ mark})$$

3. Helga should get a value of -33. She could get this value by subtracting the value of the second polynomial (26) from the value of the first polynomial (-7). (2 marks)

4. a.
$$(4n^2 - 2n - 5) + (3n^2 - 3n + 7) = 4n^2 - 2n - 5 + 3n^2 - 3n + 7$$

 $= 4n^2 + 3n^2 - 2n - 3n - 5 + 7$
 $= 7n^2 - 5n + 2$ (1 mark)

b.
$$(7+2b-3b^2)-(b^2+2b+3)+(b^2-4)=7+2b-3b^2-b^2-2b-3+b^2-4$$

= $7-3-4+2b-2b-3b^2-b^2+b^2$
= $-3b^2$ (1 mark)

c.
$$3y^2 - 7y + 4$$

 $-(y^2 + 2y)$
 $2y^2 - 9y + 4$ (1 mark)

d.
$$(7ab^2c)(2ac) = (7)(2)(a)(a)(b^2)(c)(c)$$

= $14a^2b^2c^2$ (1 mark)

e.
$$3xy(x^2 + y - 2) = (3xy)(x^2) + (3xy)(y) + (3xy)(-2)$$

= $3x^3y + 3xy^2 - 6xy$ (1 mark)

f.
$$5(x+4)-3(x+2)=5x+20-3x-6$$

= $2x+14$ (1 mark)

g.
$$\frac{4x^3y - 6x^2y + 8xy}{2xy} = \frac{4x^3y}{2xy} + \frac{-6x^2y}{2xy} + \frac{8xy}{2xy}$$
$$= 2x^2 - 3x + 4 \quad \textbf{(1 mark)}$$

h.
$$\left(4x^3y^2 - 8x^2y^3\right) + 4x^2y^2 = \frac{4x^3y^2}{4x^2y^2} + \frac{-8x^2y^3}{4x^2y^2}$$

= $x - 2y$ (1 mark)

i.
$$(2x+3)(x-4) = (2x)(x)+(2x)(-4)+(3)(x)+(3)(-4)$$

= $2x^2-8x+3x-12$
= $2x^2-5x-12$ (1 mark)

5.
$$P = 6x + (3x + 4) + 3x + 3x + 3x + 4$$

= $6x + 3x + 4 + 3x + 3x + 3x + 4$
= $18x + 8$ (1 mark)

$$A = 6x(3x+4) - 3x(3x) \text{ or } A = 4(3x) + 3x(3x+4) \text{ or } A = 6x(4) + 3x(3x)$$

$$= 18x^{2} + 24x - 9x^{2} = 12x^{2} + 9x^{2} + 12x = 24x + 9x^{2}$$

$$= 9x^{2} + 24x = 9x^{2} + 24x = 9x^{2} + 24x$$
(2 marks)

6.
$$3ab^2c$$
 (1 mark)

7. **a.**
$$4m^3 - 8m^2 + 6m = 2m(2m^2 - 4m + 3)$$
 (1 mark)

b.
$$6+18y+36y^2=6(1+3y+6y^2)$$
 (1 mark)

c.
$$4a^2 + 12a + 8 = 4(a^2 + 3a + 2)$$

= $4(a+2)(a+1)$ (1 mark)

d.
$$x^2 - 5x + 6 = (x-3)(x-2)$$
 (1 mark)

e.
$$8x^2y-12x^3y^2=4x^2y(2-3xy)$$
 (1 mark)

8. a. Like terms are terms with identical literal coefficients. (1 mark)

$$3ab^2$$
, ab^2 , and $7ab^2$ are examples of like terms. (1 mark)

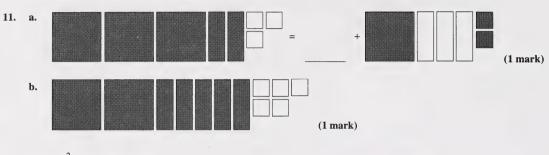
b. The degree of a polynomial is the degree of the term that has the largest sum of the exponents. (1 mark) $4m^2n+3mn+2$ is a third-degree polynomial (2+1). (1 mark)

9.
$$\frac{8a^2b^3c^4}{20ab^2c} = \frac{\frac{1}{4ab^2c}(2abc^3)}{\frac{4ab^2c}{5}}$$
$$= \frac{2abc^3}{5} \quad (1 \text{ mark})$$

10.
$$(6x^2 + 15x) \div 3x = \frac{6x^2}{3x} + \frac{15x}{3x}$$

= 2x + 5

Therefore, the length of the rectangle is 2x + 5. (1 mark)



c. $2x^2 + 5x - 5$ (1 mark)

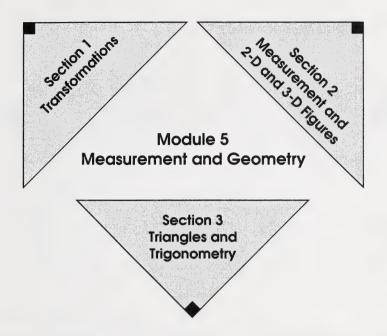
Module 5: Measurement and Geometry

Overview

This module begins with the various types of transformations on a coordinate grid. Students discover the properties of the various transformations as well as the mapping rule for each. They also discover that transformations are used in areas other than mathematics (such as in art and in advertising).

Section 2 of this module deals with spatial problems involving locus of points and geometric shapes. Students learn to draw two-dimensional views of a three-dimensional model as well as how to visualize and draw three-dimensional figures from two-dimensional plans. Students also calculate the volumes of pyramids and cones based on rectangular prisms and cylinders. In the final activity of this section students discover the relation between surface area and volume and apply this relationship to package design.

Section 3 begins with the concepts of congruent and similar triangles and what properties are required for each. The knowledge of similar triangles is extended to solve measurement problems. Activity 3 of this section uses the property of similar triangles to develop the three primary trigonometric ratios. These ratios are then used to solve numerous practical problems involving right triangles and indirect measurement. Scientific calculators are used throughout this activity.



Evaluation

The evaluation of this module will be based on four assignments:

Section 1 Assignment 25 marks
Section 2 Assignment 25 marks
Section 3 Assignment 25 marks
Final Module Assignment 25 marks

TOTAL 100 marks

Section 1: Transformations

Key Concepts

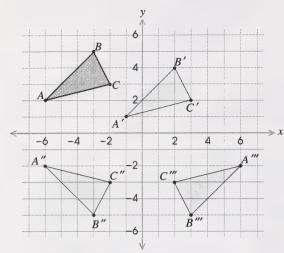
- · performing translations, rotations, reflections, and dilatations on a coordinate grid
- · representing transformations by a mapping rule
- · performing transformations from a given mapping rule

The basic goals of this section are to ensure that students are able to

- · slide, flip, turn, and dilatate figures on a coordinate grid
- · write a mapping rule for a given transformation
- · perform a transformation from a given mapping rule
- · describe the property illustrated by a given transformation

Section 1: Assignment Answer Key (25 marks)

1.



(3 marks)

2. a. Each image point is translated the same distance and direction from the corresponding point on the original figure. (1 mark)

b. Any figure and its reflection image are congruent. (1 mark)

c. A point and its $\frac{1}{2}$ -turn image are the same distance from the origin. (1 mark)

3. a. $(x,y) \rightarrow (-y,x)$ (1 mark)

b. $(x, y) \rightarrow (-x, y)$ (1 mark)

c. $(x, y) \rightarrow (x-2, y+3)$ (1 mark)

d. $(x,y) \to (2x,2y)$ (1 mark)

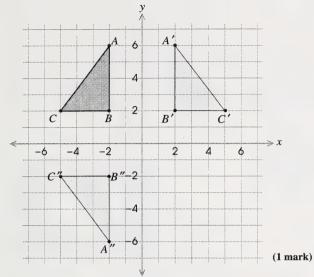
4.

Object Point	Image Point	Mapping Rule
(2,4)	(-2 , -4)	$(x,y) \rightarrow (-x,-y)$
(4, -2)	(7 ,-2)	$(x, y) \rightarrow (x+3, y)$
(8,-4)	(4,-2)	$(x,y) \rightarrow \left(\begin{array}{cc} \frac{1}{2}x & \frac{1}{2}y \end{array}\right) *$
(3,4)	(3,-4)	$(x,y) \rightarrow (x,-y)$

* An alternate solution is as follows:

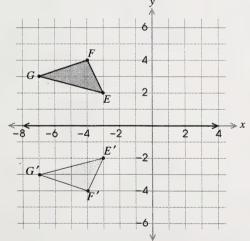
$$(x,y) \rightarrow (x-4,y+2)$$
(4 marks)

5. a.



- **b.** Refer to the answer to question 5.a. for $\Delta A'B'C'$. (1 mark)
- c. Refer to the answer to question 5.a. for $\Delta A''B''C''$. (1 mark)
- **d.** $(x,y) \rightarrow (x,-y)$ (1 mark)

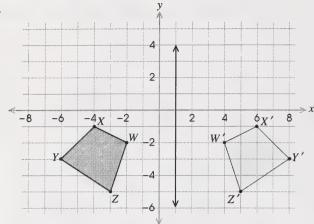
6.



(2 marks)

Note to learning facilitator: The line of reflection can be at y = 0, y = -1, y = -2, and so on. Other locations within the third quadrant for $\Delta E'F'G'$ are possible. The image is drawn for a line of reflection at y = 0.

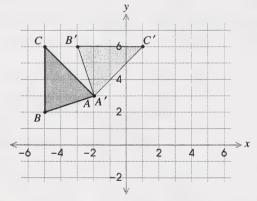
7.



(1 mark)

The coordinates of quadrilateral WXYZ are W(-2,-2), X(-4,-1), Y(-6,-3), and Z(-3,-5). The coordinates of quadrilateral W'X'Y'Z' are W'(4,-2), X'(6,-1), Y'(8,-3), and Z'(5,-5). (2 marks)

8. a



(1 mark)

b. The resulting image has the same size and shape as the original and has the same A coordinate. (1 mark)

Section 2: Measurement and 2-D and 3-D Figures

Key Concepts

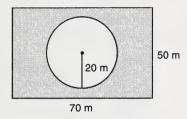
- · locus of points
- · representation of three-dimensional objects in two dimensions
- · construction of three-dimensional models from two-dimensional plans
- · volume of a pyramid based on a rectangular prism
- · volume of a cone based on a cylinder
- · effect of dimension changes on volume and surface area
- · rate of change of volume to surface area and package design

The basic goals of this section are to ensure that students

- · are able to identify a locus of points
- · are able to draw two-dimensional views of three-dimensional objects
- are able to build three-dimensional models from two-dimensional plans
- are able to find the volume of a pyramid and the volume of a cone
- understand how the volumes of pyramids and cones are related to prisms and cylinders
- · understand how changes in dimensions affect changes in surface area and volume of prisms, pyramids, and cones
- · apply rate of change of volume to surface area to package design

Section 2: Assignment Answer Key (25 marks)

1. a.



The shaded area is the area of grass the cow cannot graze. (2 marks)

b. Area not grazed = Area of field – Area grazed

$$A = \ell w - \pi r^2$$

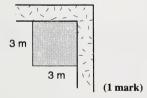
$$= 70(50) - \pi (20)^2$$

$$= 3500 - 400 \pi$$

$$= 2243.362 939$$

The area the cow cannot graze is approximately 2243 $\,\mathrm{m}^{\,2}$. (2 marks)

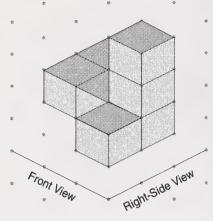
2. a. The largest area is a square.



$$b. \quad A = 3 \times 3$$
$$= 9$$

The area of the display area is 9 m². (1 mark)

3.



(3 marks)

4. **a.**
$$V = \frac{1}{3}BH$$

 $= \frac{1}{3}s^2h$
 $= \frac{1}{3}(12)^2(18)$
 $= \frac{1}{3}(144)(18)$
 $= 864 \text{ cm}^3$ (2 marks)

b.
$$V = \frac{1}{3}BH$$

 $= \frac{1}{3} \left(\frac{nsa}{2}\right)h$
 $= \frac{1}{3} \left(\frac{6(8)(6.9)}{2}\right)(24)$
 $= \frac{1}{3}(165.6)(24)$
 $= 1324.8 \text{ cm}^3$ (2 marks)

5. The volume will be 2^3 or 8 times greater. (1 mark)

6. **a.**
$$V = \frac{1}{3}BH$$

 $= \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi (9)^2 (21)$
 $= \frac{1}{3}\pi (81)(21)$
 $= 1781.283 035$

The volume of the cone is approximately 1781 $\,\mathrm{cm}^{\,3}$. (2 marks)

b.
$$1781 \times \left(\frac{1}{3}\right)^3 \doteq 1781 \left(\frac{1}{27}\right)$$

 $\doteq 65.962\ 962\ 96$

The volume of the cone will be about 66 cm³. (1 mark)

7. $24 \times 3 = 72$

The volume of the cylinder is 72 cm³. (1 mark)

8. If you double the length of the sides of a cube, then its surface area increases 4 times and its volume increases 8 times. (2 marks)

9. a.
$$\frac{V \text{ of container}}{V \text{ of boxes}} = \frac{24 \times 24 \times 24}{6 \times 6 \times 6}$$
$$= \frac{13824}{216}$$
$$= 64$$

b. $\frac{V \text{ of container}}{V \text{ of boxes}} = \frac{36 \times 36 \times 36}{6 \times 6 \times 6}$ $= \frac{46656}{216}$ = 216

The container will hold 64 boxes. (1 mark)

The container will hold 216 boxes. (1 mark)

10. SA of smaller container =
$$6s^2$$

= $6(24)^2$
= $6(576)$
= 3456 cm^2

SA of larger container =
$$6s^2$$

= $6(36)^2$
= $6(1296)$
= 7776 cm^2

$$\therefore \frac{3456}{64} = 54 \text{ cm}^2 \text{ per box shipped}$$
(1 mark)

$$\therefore \frac{7776}{216} = 36 \text{ cm}^2 \text{ per box shipped}$$
(1 mark)

The larger container is more cost-effective. In terms of surface area, it uses 36 cm² of material per box shipped while the smaller container uses 54 cm² of material per box shipped. (1 mark)

Section 3: Triangles and Trigonometry

Key Concepts

- · triangles can be congruent
- · triangles can be similar
- trigonometric ratios for an angle in a right triangle can be calculated
- a right triangle can be solved for its parts
- · trigonometric ratios can be used in problem solving

The basic goals of this section are to ensure that students

- · understand what conditions are required to make triangles congruent
- · understand what conditions are required to make triangles similar
- · recognize and understand the three primary trignometric ratios
- are able to use the ratios to find the measures of missing angles and sides in right triangles
- are able to apply the trigonometric ratios to a variety of situations to solve problems

Section 3: Assignment Answer Key (25 marks)

1. a.
$$\Delta PAT \cong \Delta DOG$$
 (1 mark)

c.
$$\angle O = 90 - 32$$

= 58° (1 mark)

d.
$$PA \cong DO$$
 $\angle P \cong \angle D$
 $AT \cong OG$ $\angle A \cong \angle O$
 $PT \cong DG$ $\angle T \cong \angle G$
(1 mark) (1 mark)

2. **a.**
$$\frac{4}{10} = \frac{5.4}{n+5.4}$$
 (1 mark)

Note to learning facilitator: The ratios of the corresponding sides may vary.

b.
$$4 \times (n+5.4) = 5.4 \times 10$$

 $4n+21.6 = 54$
 $4n+21.6-21.6 = 54-21.6$
 $4n=32.4$
 $\frac{4n}{4} = \frac{32.4}{4}$
 $n=8.1 \text{ cm} (1 \text{ mark})$

- 3. Any two of the following statements will be sufficient.
 - ΔABC and ΔDEF are similar.

•
$$\angle A = \angle D$$
, $\angle B = \angle E$, and $\angle C = \angle F$.

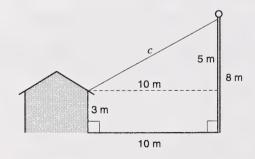
- · The corresponding sides of the two triangles are proportional.
- The statement says the ratio of any two sides of one triangle is equal to the ratio of the two corresponding sides of the other triangle.

(2 marks)

4.
$$c^2 = a^2 + b^2$$

 $c^2 = 10^2 + 5^2$
 $c^2 = 100 + 25$
 $c^2 = 125$
 $c = \sqrt{125}$
 $c = 11.18033989$

The wire is about 11.2 m long. (2 marks)



5. **a.**
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$
 $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

$$= \frac{2.5}{6.5}$$

$$= \frac{5}{13}$$

$$= \frac{0.385}{0.385}$$

(1 mark)

6. Method 1



Method 2

 $\therefore \angle \theta \doteq 23^{\circ}$ (2 marks)

7.
$$a^{2} = c^{2} - b^{2}$$

$$a^{2} = (74)^{2} - (32)^{2}$$

$$a^{2} = 5476 - 1024$$

$$a^{2} = 4452$$

$$a = \sqrt{4452}$$

$$a = 66.72330927$$

Thus, BC is about 67 cm. (1 mark)

$$\sin B = \frac{\text{opp}}{\text{hyp}}$$

$$\sin B = \frac{32}{74} \text{ (or cos } A\text{)}$$

$$B \doteq 25.622 \text{ } 027.75$$

 $\therefore \angle B$ is about 26° (1 mark)

$$\angle A \doteq 90^{\circ} - 26^{\circ}$$

 $\doteq 64^{\circ}$ (1 mark)

8.
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

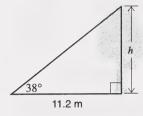
$$\cos \theta = \frac{24}{40}$$

$$\cos \theta = 0.6$$

 $\therefore \angle \theta \doteq 53^{\circ}$ (1 mark)

$$\theta = 53.130\,102\,35$$

9.



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 38^\circ = \frac{h}{11.2}$$

$$h = 11.2 \times \tan 38^{\circ}$$

Therefore, the tree is about 8.8 m tall. (2 marks)

10.
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 75^\circ = \frac{30}{c}$$

$$c = \frac{30}{\sin 75^{\circ}}$$

Thus, each cable should be about 31.1+1=32.1 m long. (2 marks)

11.
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 75^\circ = \frac{30}{b}$$

$$b = \frac{30}{\tan 75^{\circ}}$$

The cables are fastened to the ground about 8.0 m from the centre of the tower. (1 mark)

Note: Students may also use the Pythagorean Theorem.

12.



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

or
$$\tan \theta = \frac{4.2}{15.6}$$

$$\tan 14^\circ = \frac{4.2}{d}$$

$$\tan \theta \doteq 0.269 \ 230 \ 769$$

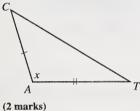
 $\theta \doteq 15.068 \ 488 \ 16$

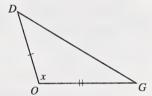
$$d = \frac{4.2}{\tan 14^{\circ}}$$

No, the airplane needs to increase its angle of descent by about 1° or it will overshoot the start of the runway by about 1.2 km. (2 marks)

Final Module Assignment Answer Key (25 marks)

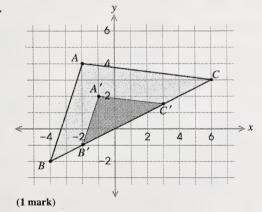
1. Triangles may be different than the ones shown.





(2 marks

2. a.

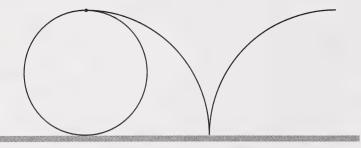


b. The scale factor is $\frac{1}{2}$. (1 mark)

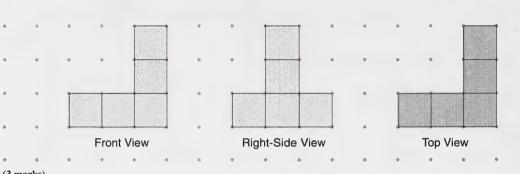
c.
$$A'B' = 4.8 \times \frac{1}{2}$$

= 2.4 cm (1 mark)

3.



(2 marks)



(3 marks)

5.
$$\left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

The volume of the cube is reduced by a factor of $\frac{1}{64}$. (1 mark)

- Yes, the two cylinders have the same surface area since both are made from the same size net. (1 mark)
 - Answers will vary. Students may predict either to have the larger volume. (1 mark) b.
 - The volume of the cylinder with the smaller radius is approximately 1000 cm³. The volume of the cylinder with the larger radius is approximately 1300 cm^3 . (Remember: $1 \text{ mL} = 1 \text{ cm}^3$) (1 mark)
 - The circumference of one cylinder is equal to the length of the sheet of paper while the circumference of the other is equal to the width.

Cylinder with Smaller Radius

$$C = 2\pi r$$

$$r = \frac{C}{2\pi}$$

$$= \frac{21.5}{2\pi}$$

$$= 3.421831276$$

$$\therefore V = \pi r^2 h$$

$$\stackrel{\cdot}{=} \pi (3.42)^2 \times 28$$

$$\stackrel{\cdot}{=} 1029$$

The volume is approximately 1029 cm. (1 mark)

The radius is about 3.42 cm. (1 mark)

Cylinder with Larger Radius

$$C = 2\pi r$$

$$r = \frac{C}{2\pi}$$

$$= \frac{28}{2\pi}$$

$$= 4.456338407$$

$$\therefore V \doteq \pi r^2 h$$

$$\doteq \pi (4.46)^2 \times 21.5$$

$$\doteq 1312$$

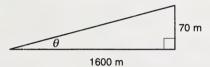
The volume is approximately 1312 cm³. (1 mark)

The radius is about 4.46 cm. (1 mark)

The volumes calculated should be close to the volumes measured.

e. The cone with the larger radius would have the greater volume.

7.

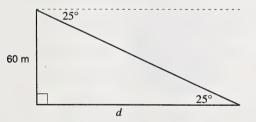


$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$
$$= \frac{70}{1600}$$



The track rises at an angle of about 2.5°. (3 marks)

8.



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 25^\circ = \frac{60}{d}$$
$$d = \frac{60}{d}$$



The river is about 129 m wide. (2 marks)

Module 6: Data Management

Overview

This module begins with examples of bivariate data and explores relationships between variables by conducting experiments. Students analyse how to design and conduct an experiment and gather data in order to determine what kind of relationship might exist between two variables. They then display this data on a scatter plot. Students complete this section by drawing a line of best fit and using it to predict outcomes.

Section 2 assesses the strengths, weaknesses, and biases of data-collection methods by analysing several newspaper articles and advertisements involving data. Students then examine the various ways in which information is presented and watch for false impressions and exaggerations.

Section 3 looks at theoretical and experimental probability and their roles in today's world. Students are shown that many everyday decisions and actions are based on probability as well as subjective judgements. Students learn to calculate the probability of independent events and apply this to solving problems. Students have the opportunity to use a computer spreadsheet to simulate probability experiments involving independent events.

Module 6
Data Management

Section 1
Bivariate Data
and
Scatter Plots

Section 2
Use and
Misuse
of Data
Decision
Making

Evaluation

The evaluation of this module will be based on four assignments:

Section 1 Assignment
Section 2 Assignment
Section 3 Assignment
Final Module Assignment

25 marks
15 marks
20 marks
40 marks

TOTAL 100 marks

Section 1: Bivariate Data and Scatter Plots

Key Concepts

- · designing and conducting experiments involving bivariate data
- · creating and analysing scatter plots
- · drawing lines of best fit
- · drawing and justifying conclusions from lines of best fit

The basic goals of this section are to ensure that students are able to

- · design and carry out experiments to investigate relationships between two variables
- · draw scatter plots and analyse them for an apparent relationship
- draw lines of best fit on scatter plots to show a possible linear relationship
- · use the lines of best fit to draw conclusions or make predictions

Section 1: Assignment Answer Key (25 marks)

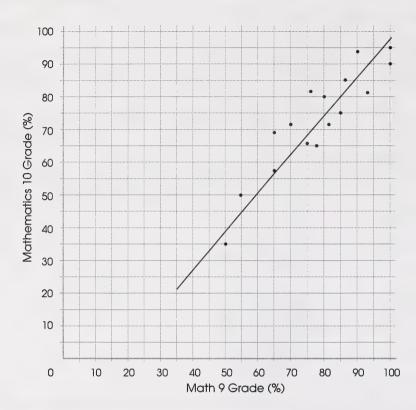
• a. The following is a suggested design for carrying out this experiment.

You could use a computer simulation that would measure the reaction time for a driver. You could have a number of drivers and test their reaction time after drinking various amounts of alcohol over prescribed time periods. (2 marks)

- b. amount of alcohol (mL) and reaction time (seconds) (1 mark)
- c. Answers will vary. Some factors are as follows:
 - · age of drivers
 - · amount of sleep drivers had
 - · weight of drivers
 - · when they last had something to eat
 - · general health of drivers
 - · time between drinks

(2 marks)

Math 10 Grade Versus Math 9 Grade



Note: The line of best fit does not have to be exactly as shown.

(2 marks for scatter plot; 1 mark for line of best fit)

- b. Students who do well in Math 10 would generally have a high Math 9 mark. (1 mark)
- c. Credits in Math 9 (50% or higher) are needed to take the Math 10 course. (1 mark)
- d. Yes, if you have a good mark in Math 9, you most likely will get a good mark in the course. (1 mark)
- 3. a. There does appear to be a relationship with longer cats having larger masses; but the relationship is not very distinct. (2 marks)
 - b. The data involving the length could have just included body length. The length of the tail varies considerably and adds very little to the mass of the cat. (2 marks)
- **4.** a. Time is the dependent variable. $\left(\frac{1}{2} \text{ mark}\right)$

Distance is the independent variable. $\left(\frac{1}{2} \text{ mark}\right)$

The points will rise to the right. (1 mark)

2.

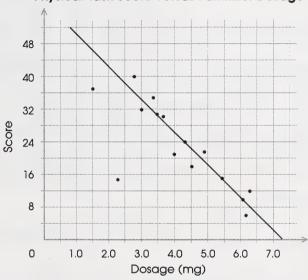
b. The percentage of shots made is the dependent variable. $\left(\frac{1}{2} \text{ mark}\right)$

The distance from the basket is the independent variable. $\left(\frac{1}{2}\ mark\right)$

The points will decrease to the right. (1 mark)

5. a. The following is a sample scatter plot with line of best fit. The position of the line of best fit will vary somewhat but should be close to the middle of the majority of points.





(2 marks)

- b. As the drug dosage increases, the physical capabilities decrease. (1 mark)
- c. A score of about 32 would be expected. (1 mark)
- d. There would be no effect up to a dosage of about 0.80 mg. (1 mark)
- e. A score of about 2 might be expected for a dosage of 7 mg. (1 mark)

Section 2: Use and Misuse of Data

Key Concepts

- assessing strengths, weaknesses, and biases of data-collection methods
- · critiquing ways in which media and industry use data

The basic goals of this section are to ensure that students

- · are able to analyse the various ways of collecting data to avoid bias, and to learn which method might suit their specific needs
- are aware of the ways in which statistical information and conclusions are presented by the media and other sources, and know what to look for

Section 2: Assignment Answer Key (15 marks)

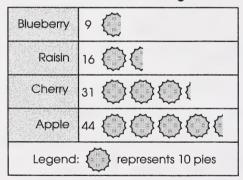
1. The part of the question "a healthy body promotes a healthy mind" influences the person answering the part of the question that physical education is beneficial.

It would be better written as follows:

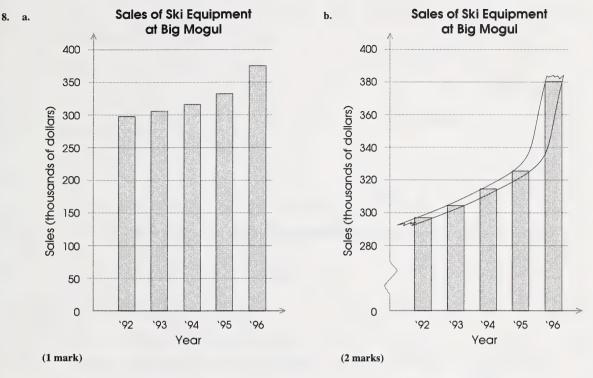
Should physical education be compulsory? (3 marks)

- 2. The data was gathered by surveying 703 Albertans. It is not stated whether the survey was through telephone, personal interview, or questionnaire. (1 mark)
- 3. The conclusion indicated which federal political party Albertans supported the most. Since only Albertans were surveyed, the conclusion is appropriate. (2 marks)
- 4. The information appears to be presented without bias; but there seems to be some clouding of the main issue with a second poll regarding the main campaign issues on the minds of voters. (1 mark)
- 5. Yes, the question regarding the main campaign issues is asked clearly and without bias. (1 mark)
- 6. According to the numbers, apple pie filling was preferred about 5 times as much as blueberry; but the area of the pie for apple is about 25 times the area of the pie picture for blueberry. (2 marks)
- 7. The following is one type of non-biased graph which could be drawn. Other graphs may be acceptable.

Preferred Pie Filling



(2 marks)



Section 3: Probability and Decision Making

Key Concepts

- · calculating the probability of several independent events from the product of the probability of each event
- · using probability of independent events to solve problems
- · using a combination of theoretical calculations, experimental results, and subjective judgements to make decisions
- · calculating the probability of certain numbers coming up in a lottery based on probability
- importance of probability in weather forecasting

The basic goals of this section are to ensure that students

- · are able to calculate the probability of independent events
- are able to solve problems involving probability of independent events
- · understand that a decision may be the result of a subjective judgement rather than a theoretical calculation or an experimental result
- understand that decisions to play certain lottery numbers or not to participate in a particular activity upon hearing the weather forecast are subjective
- · understand that probability plays a role in our everyday lives

Section 3: Assignment Answer Key (20 marks)

1.
$$P(sum \ of \ 3) = \frac{1}{18} \left(\frac{1}{2} \ mark\right)$$

$$P(black disk) = \frac{4}{10}$$
$$= \frac{2}{5} \left(\frac{1}{2} \text{ mark}\right)$$

$$P(sum \ of \ 3 \ and \ black \ disk) = \frac{1}{18} \times \frac{2}{5}$$

= $\frac{1}{45}$ (1 mark)

The probability of getting a sum of 3 when rolling two dice and picking a black disk is $\frac{1}{45}$. (1 mark)

2. a.
$$P(even \ and \ grey) = \frac{1}{2} \times \frac{1}{2}$$

= $\frac{1}{4}$ (1 mark)

b. $P(greater than 1 and white) = \frac{5}{6} \times \frac{1}{2}$ = $\frac{5}{12}$ (1 mark)

3. **a.**
$$P(6H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{64}$$
(1 mark)

b. The probability of it coming up heads on the seventh toss is $\frac{1}{2}$. The only consideration is that it can come up one of two ways. The previous six tosses have no effect on the seventh toss. (2 marks)

4. a.
$$P(Monday \ and \ Wednesday) = 0.70 \times 0.40$$

= 0.28

The probability of rain on Monday and Wednesday is 28%. (2 marks)

b.
$$P(rain five days) = 0.70 \times 0.90 \times 0.40 \times 0.20 \times 0.30$$

= 0.015 12

The probability of rain on all five days is about 1.5%. (2 marks)

c. Bernice should go ahead with her plans for the weekend. She will base her decision on the fact that the chances of rain on the last two days before the weekend are quite low. (2 marks)

- 5. It is appealing because the cost of a ticket is low (\$1) and the prize that could be won is large. (1 mark)
- **6. a.** The possible sums are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

The prime numbers are 2, 3, 5, 7, and 11.

$$P(Player\ 1) = \frac{5}{11}$$

The composite numbers are 4, 6, 8, 9, 10, and 12.

$$P(Player 2) = \frac{6}{11}$$

(2 marks)

- b. The game is not fair since there are more chances for Player 2 than for Player 1. (1 mark)
- c. You can make the game fair by eliminating one of the composite numbers, say the number 12.

$$P(Player 1) = \frac{5}{10}$$
 $P(Player 2) = \frac{5}{10}$ (2 marks)

Final Module Assignment Answer Key (40 marks)

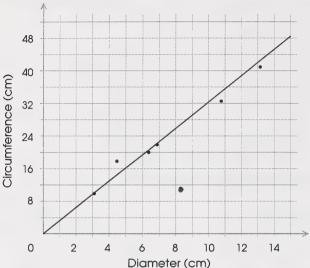
- a. Their claim is not entirely valid since the price has only dropped approximately 10%. (1 mark)
 - b. The vertical axis is stretched out to make the drops look significant, and the bottom of the graph is omitted. (1 mark)
- a. The radio station is trying to persuade more people to listen to them because they are more popular. (1 mark)
 - b. You can conclude that at least one more person is listening than ever before. (1 mark)
 - c. Answer may vary. Some sample answers are given:
 - · number of listeners over the past two or three years versus number of listeners now
 - number of listeners of competing stations

(1 mark)

- 3. a. Weight increases as height increases. (1 mark)
 - b. Flexibility decreases as age increases. (1 mark)
 - c. There would be no apparent relation between hours of sleep and calorie intake. (1 mark)

4. a. Your scatterplot should be similar to the following:

Circumference Versus Diameter of Certain Objects



(4 marks)

b. You would expect a circumference of about 32 cm. (1 mark)

c.
$$C = \pi d$$

$$\pi = \frac{C}{d}$$

$$=\frac{32}{}$$

$$= 3.2$$

(1 mark)

Yes, the value of π is reasonable. (1 mark)

- d. The value could be off due to an error in measuring or in reading the measurement. (1 mark)
- 5. a. The machine is 30% better than an earlier model.
 - The machine is 30% better than a rival machine. (1 mark)
 - b. The statement doesn't say what it is 30% better than, and doesn't define what it means by "better." (1 mark)
- 6. a. In the company profits graph, the dollar sign gets fatter as well as taller. (1 mark)

In the sales graph, the bottom of the bars have been cut off. (1 mark)

b. The fatter dollar sign makes it look like company profits have at least doubled when in fact they have only gone up about 35% (\$300 000 \rightarrow \$400 000). (1 mark)

It appears that Company B is double Company C in sales and almost double Company A, when in fact is only 8% and $6\frac{1}{2}$ % higher. (1 mark)

 The company profits graph could be modified by using plain bars or by using the appropriate number of equal sized dollar signs. (1 mark)

The sales graph could be modified by starting the bars from zero. (1 mark)

- 7. **a.** $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ (1 mark)
 - **b.** $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{64}$ (1 mark)
 - c. It is assumed that the probability of winning or losing is the same for each team in each game. (1 mark)
- 8. a. III (1 mark)
 - b. Statement I is biased towards gun owners. (1 mark)

Statement II is biased toward people who do not own guns, people who have been the victims of guns, or people who just don't think guns are necessary. (1 mark)

9.
$$\underbrace{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times ... \times \frac{1}{2}}_{10 \text{ times}} = \frac{1}{1024}$$
 (2 marks)

10. Answers will vary according to the article selected. Answers should refer to the article and be in complete sentences. Refer to answers for similar articles in Section 2, Activity 1 of the Student Module Booklet. (6 marks)

11. **a.**
$$P(rain) = 0.70 \times 0.60 \times 0.20$$

= 0.084

The chance of rain on all three days is about 8%. (1 mark)

- b. No, the answer to question 11.a. means that you will not likely have rain on all three days. (1 mark)
- c. You may decide to go anyway since part of Saturday and all of Sunday could possibly be without rain. You may not want to go at all since the ground could be wet all weekend. (1 mark)



Final Test

Security

Included here is the answer key to the Final Test and the student's copy of the Final Test. Teachers should keep these secure against unauthorized student access. Students should not have access to the test until it is assigned in a supervised situation. The answers should be stored securely and retained by the teacher at all times.

Convenience

The student's copy of the Final Test is designed for photocopying and faxing.

MATHEMATICS 9

FINAL TEST ANSWER KEY

Part A: Number (25 marks)

- 1. The numbers listed could all belong to the set of rational numbers. (1 mark)
- 2. $\sqrt{5.0 \times 10^9} = 7.1 \times 10^4$ (1 mark)
- 3. π is not a rational number because it is a nonrepeating, nonterminating decimal which cannot be put in the form of $\frac{a}{b}$. (2 marks)
- 4. $\frac{3(2)^{2}(-4)}{-2(-3)} = \frac{3(4)(-4)}{-2(-3)}$ $= \frac{-48}{6}$ $= -8 \quad (2 \text{ marks})$
- 5. 9 2 + 6 4 = × 2 2 1 = Min

 2 8 7 1 1 4 = + MR =

 (2 marks)
- 6. The method in question 5 is neither more or less efficient. Both methods require the same number of keystrokes. (1 mark)
- 7. Answers may vary. One advantage is that if you make an error after you have stored part of the answer, then you need only enter the keystrokes after the memory key was used. (1 mark)
- 8. Method 1



Method 2



(2 marks)

Hillary would receive \$5.06 in change. (1 mark)

9.
$$2^{1} = 2$$
 $2^{5} = 32$ $2^{6} = 64$ $2^{3} = 8$ $2^{7} = 128$ $2^{8} = 256$

The cycles repeat in groups of four (last digit is 2, 4, 8, 6). To find the last digit of 2^{1003} , you must go through 250 complete cycles and three steps into the 251st cycle. Therefore, 2^{1003} ends in 8. (2 marks)

10.
$$\frac{\left(3.2 \times 10^{12}\right) \times \left(1.5 \times 10^{-8}\right)}{\left(2.4 \times 10^{8}\right) \times \left(1.6 \times 10^{5}\right)} = \frac{(3.2)(1.5)\left(10^{12}\right)\left(10^{-8}\right)}{(2.4)(1.6)\left(10^{8}\right)\left(10^{5}\right)}$$
$$= \frac{4.8 \times 10^{4}}{3.84 \times 10^{13}}$$
$$= 1.25 \times 10^{-9} \quad (2 \text{ marks})$$

11.
$$\frac{2^9 \times 3^{13} \times 4^2}{2^6 \times (3^2)^4 \times 4} = 2^{9-6} \times 3^{13-8} \times 4^{2-1}$$
$$= 2^3 \times 3^5 \times 4$$
$$= 7776 \quad (2 \text{ marks})$$

12. **a.**
$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{5}{6} = \frac{1}{2} + \frac{1}{3} \times \frac{2}{1} + \frac{5}{6}$$

$$= \frac{1}{2} + 2 + \frac{5}{6}$$

$$= \frac{3}{6} + \frac{12}{6} + \frac{5}{6}$$

$$= \frac{20}{6}$$

$$= \frac{10}{3} \text{ or } 3\frac{1}{3} \text{ (1 mark)}$$

b.
$$4 - \left[\left(\frac{3}{8} + \frac{1}{2} \right) \div \frac{1}{4} \right] = 4 - \left[\left(\frac{3}{8} + \frac{4}{8} \right) \div \frac{1}{4} \right]$$

$$= 4 - \left(\frac{7}{8} \div \frac{1}{4} \right)$$

$$= 4 - \left(\frac{7}{8} \times \frac{\frac{1}{4}}{1} \right)$$

$$= 4 - \left(\frac{7}{2} \right)$$

$$= \frac{8}{2} - \frac{7}{2}$$

$$= \frac{1}{2} \quad (1 \text{ mark})$$

c.
$$0.48 + 6.30 - 9.77 = -2.99$$
 (1 mark)

d.
$$2^{-3} \times 2^{5} + \left[3^{2} + \left(\frac{1}{3}\right)^{-3}\right] \div 2^{2} = \frac{1}{8} \times 32 + (9 + 27) \div 4$$

$$= \frac{1}{8} \times 32 + 36 \div 4$$

$$= 4 + 36 \div 4$$

$$= 4 + 9$$

$$= 13 \quad (1 \text{ mark})$$

13.
$$4.0176 \times 10^{13} \div 1.08 \times 10^{9} = 3.72 \times 10^{4}$$

 $3.72 \times 10^{4} \div 24 = 1550$

The light will take 1550 days to reach Earth. (2 marks)

Part B: Patterns and Relations (30 marks)

1.
$$5(n+1) = 2(n+4)$$

 $5n+5 = 2n+8$
 $5n+5-5 = 2n+8-5$
 $5n = 2n+3$
 $5n-2n = 2n+3-2n$
 $3n = 3$
 $\frac{3n}{3} = \frac{3}{3}$
 $n = 1$ (2 marks)

2. 2b-5 < 20 (2 marks)

3.
$$5a-3=2a+3$$

$$5a-3+3=2a+3+3$$

$$5a = 2a + 6$$

$$5a-2a=2a+6-2a$$

$$3a = 6$$

$$\frac{3a}{3} = \frac{6}{3}$$

$$a=2$$
 (2 marks)

- 4. Step 1: Represent the algebraic equation by placing two positive cylinders and three negative counters on one side of the balance and one negative cylinder and six positive counters on the other side.
 - Step 2: Add a positive cylinder to each side, and remove the zero pair.
 - Step 3: Add three positive counters to each side, and remove the zero pairs.
 - **Step 4:** Divide the cylinders and counters into three groups, and determine the number of counters in one group. This is the solution to the equation.

(4 marks)

5.
$$v = \sqrt{19.6 h}$$

$$v^2 = 19.6h$$

$$v = 19.0 n$$

$$\frac{v^2}{19.6} = \frac{19.6 \, h}{19.6}$$

$$\frac{v^2}{19.6} = h$$

$$h = \frac{v^2}{19.6}$$

$$=\frac{(5)^2}{19.6}$$

$$=\frac{25}{19.6}$$

$$= \frac{1}{19.6}$$
$$= 1.275510204$$

The height of the liquid above the hole is approximately 1.3 m. (2 marks)

6. This problem may be done several ways. One method is shown.

Let the time the second cyclist is travelling be n. Let the time the first cyclist is travelling be n+2.

•	Time	Speed	Distance
Cyclist #1	n+2	12	12(n+2)
Cyclist #2	n	18	18 <i>n</i>

The cyclists travel the same distance. Write an equation to show this.

$$18n = 12(n+2)$$

$$18n = 12n + 24$$

$$18n - 12n = 12n + 24 - 12n$$

$$6n = 24$$

$$\frac{6n}{6} = \frac{24}{6}$$

$$n = 4$$

It takes the second cyclist 4 h to catch up to the first cyclist. (3 marks)

7. Answers will vary. A sample answer is given.

a.
$$4a^2b + 9b$$
 (1 mark)

b.
$$2xy^2z - 3xy + 5$$
 (1 mark)

8.
$$-2x^2 + 2x + 4$$
 (1 mark)

9.
$$2(-2)(3)^2 - 4(-2) + 7 = 2(-2)(9) - 4(-2) + 7$$

= $-36 + 8 + 7$
= -21 (2 marks)

10.



(1 mark)

11. **a.**
$$(2a^3 + 5a^2 - 4) - (a^3 - 2a^2 - 3) = 2a^3 + 5a^2 - 4 - a^3 + 2a^2 + 3$$

 $= 2a^3 - a^3 + 5a^2 + 2a^2 - 4 + 3$
 $= a^3 + 7a^2 - 1$ (1 mark)

b.
$$\frac{12n^3 - 18n^2 - 9n}{3n} = \frac{12n^3}{3n} + \frac{-18n^2}{3n} + \frac{-9n}{3n}$$
$$= 4n^2 - 6n - 3 \quad (1 \text{ mark})$$

12.
$$7x-9-(x-3)=7x-9-x+3$$

= $6x-6$

The measure of the two equal sides is 6x-6.

$$\frac{6x-6}{2} = 3x-3$$

The expression for each of the missing dimensions is 3x-3. (2 marks)

13. a.
$$4xy^2 - 8x = 4x(y^2 - 2)$$
 (1 mark)

b.
$$3x^2 - 9x + 6 = 3(x^2 - 3x + 2)$$

= $3(x-2)(x-1)$ (1 mark)

c.
$$a^3 - 3a^2 - 10a = a(a^2 - 3a - 10)$$

= $a(a-5)(a+2)$ (1 mark)

14. a.
$$(n-6)(n+7) = (n)(n)+(n)(7)+(-6)(n)+(-6)(7)$$

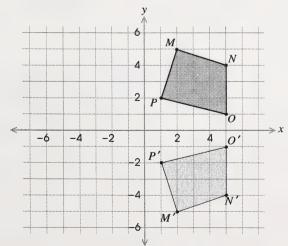
= $n^2 + 7n - 6n - 42$
= $n^2 + n - 42$ (1 mark)

b.
$$(4a+3)(4a+3) = (4a)(4a) + (4a)(3) + (3)(4a) + (3)(3)$$

= $16a^2 + 12a + 12a + 9$
= $16a^2 + 24a + 9$ (1 mark)

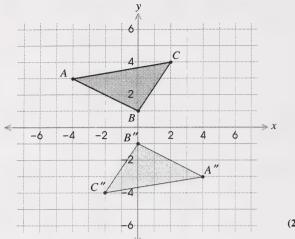
Part C: Shape and Space (25 marks)

- 1. a. The transformation that is illustrated is that of a translation. (1 mark)
 - **b.** $(x,y) \rightarrow (x-2,y-3)$ (2 marks)
- 2. a. Answers will vary. The line of reflection could be the x-axis, y = -1, y = -2, and so on. (1 mark)
 - **b.** A possible line of reflection and image for quadrilateral MNOP is shown on the following grid. (**Note:** The line of reflection is the x-axis.)



(1 mark)

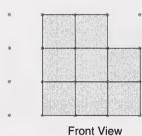
3.



(2 marks)

The coordinates of $\Delta A'B'C'$ are A'(4,3), B'(0,1), and C'(-2,4). The coordinates of $\Delta A''B''C''$ are A''(4,-3), B''(0,-1), and C''(-2,-4). (2 marks)

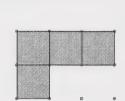
4. »



Front view

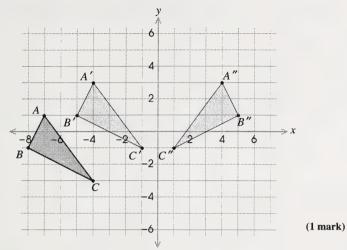
(3 marks)

Right-Side View



Top View

5.

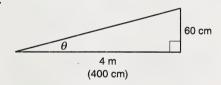


Note: $\Delta A'B'C'$ is the position of the triangle before the reflection.

The coordinates of $\triangle ABC$ are A(-7,1), B(-8,-1), and C(-4,-3). (1 mark)

6. Yes, $\triangle ABC \cong \triangle DEF$. Two corresponding sides and the contained angle (SAS) of the two triangles are congruent. Thus, you can conclude that the two triangles are congruent and that all corresponding sides and angles are congruent. (2 marks)

7.



$$\tan \theta = \frac{60}{400}$$
$$= \frac{3}{20}$$



Therefore, the ramp rises at an angle of about 8.5°. (3 marks)

8. Volume of Pyramid

$$V = \frac{1}{3}BH$$

$$= \frac{1}{3}s^{2}h$$

$$= \frac{1}{3}(15)^{2}(20)$$

$$= \frac{1}{3}(225)(20)$$

$$= 1500 \text{ cm}^{3}$$

$$V = \frac{1}{3}BH$$

$$= \frac{1}{3}\pi r^{2}h$$

$$= \frac{1}{3}\pi (7.5)^{2}(20)$$

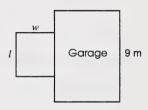
$$= \frac{1}{3}\pi (56.25)(20)$$

$$= 1178 \text{ cm}^{3}$$

The pyramid-shaped container is the better buy for the consumer. (3 marks)

9. Using a chart, try different widths and lengths for the enclosure and find the largest area.

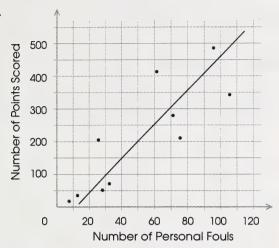
Length (m)	Width (m)	Area (m²)
12	4	48
10	5	50
8	6	48
6	7	42



The length of the fence cannot be greater than 9 m (the length of the garage). Therefore, the greatest area that can be enclosed is 48 m^2 . (3 marks)

Part D: Statistics and Probability (20 marks)

l. a



(2 marks)

- b. The greater the number of personal fouls, the more points the player scored. (1 mark)
- c. Refer to the answer to question 1.a. for the line of best fit. Note: The line of best fit may vary a little (but not too much) from the one given. (1 mark)
- d. The player will score about 250 points. (1 mark)
- e. The prediction is not very accurate because the data shows a player with 61 personal fouls who scored 414 points. (1 mark)
- The greater the number of passes completed, the higher the team's standing will be at the end of the year. (1 mark)
- 3. This graph is misleading in that although sales in September and October are about twice and three times those in August, the pictures are about four times and nine times larger than the one in August. This implies a much larger sales amount in September and October as compared to August than there actually is. (3 marks)
- 4. $P(H, H, H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ = $\frac{1}{8}$ (2 marks)
- 5. Over the long term, all numbers picked for a Lotto 6/49 draw have an equal chance of coming up. Therefore, numbers picked based on numbers that came up previously would have the same chance of coming up again as any other number. (2 marks)
- 6. It is true that the murder rate did drop by over 50%, but this statistic is only for one year. The headline implies that the rate of murder over a period of time has dropped. This may not be the case. Next year the murder rate may be back up to 20. (2 marks)
- 7. **a.** $0.60 \times 0.40 \times 0.30 = 0.072$

$$\doteq 0.07$$
 or 7% (2 marks)

b. The probability calculated in question 7.a. is so low because it is the probability that it will rain on all three days, not just whether it will rain on any one day. (2 marks)



MATHEMATICS 9

FINAL TEST

GENERAL INSTRUCTIONS

YOU HAVE TWO HOURS TO COMPLETE THIS TEST. Work through the entire test answering the questions you are sure you know. You will then be able to concentrate on the questions of which you are not quite sure.

TOTAL MARKS: 100

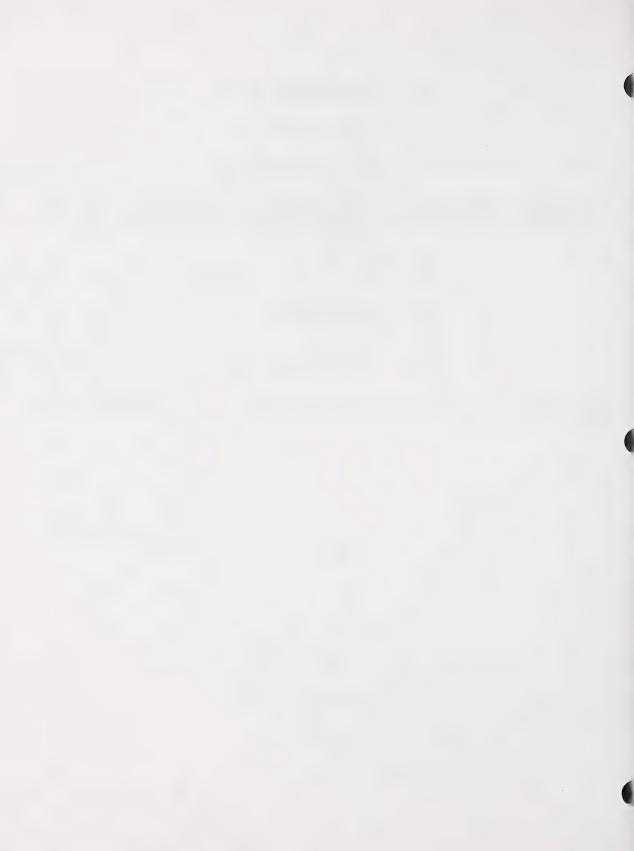
PART A: Number 25 marks

PART B: Patterns and Relations 30 marks

PART C: Shape and Space 25 marks

PART D: Probability and Statistics 20 marks

Note: You will need a scientific calculator and a mathematical instrument set. Also, a formula sheet has been provided at the back of this test.



Value

PART A: NUMBER

25

Answer each of the following questions using the space provided. Clearly show how you arrived at your answers.

(1 mark)

1. The following numbers could all belong to which one group of numbers?

$$3, -1, \frac{-7}{1}, \frac{4}{5}, 0.27$$

(1 mark)

2. Determine the square root of 5.0×10^9 to one decimal place. Write the answer in scientific notation.

(2 marks)

3. Explain why π is not a rational number.

(2 marks)

4. Determine the value of $\frac{3a^2c}{-2b}$ when a=2, b=-3, and c=-4.

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(1 mark)

Use the following information to answer questions 5, 6, and 7.

Allan used a calculator to calculate the answer to the following expression.

$$\frac{28.7 - 11.4}{22.1(9.2 + 6.4)}$$

The keystrokes he used are as follows:



The answer he obtained was 0.05, rounded to two decimal places.

(2 marks) 5. Devise a method to perform this calculation using the memory keys.

(1 mark) 6. Is the method in question 5 more or less efficient than the method Allan used?

7. What might be an advantage of using the memory key over using brackets?

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(3 marks)

- 8. Hillary buys the following items of clothing.
 - 2 blouses @ \$17.99 each
 - 2 scarfs @ \$6.49 each
 - 2 pair of slacks @ \$22.99 each

How much change will Hillary receive from a \$100 bill? Write out the keystrokes you would use to perform this calculation in one sequence.

(2 marks)

9. Determine the last digit when 2^{1003} is written in standard form.

(2 marks)

10. Calculate the following. Write your answer in scientific notation.

$$\frac{\left(3.2 \times 10^{12}\right) \times \left(1.5 \times 10^{-8}\right)}{\left(2.4 \times 10^{8}\right) \times \left(1.6 \times 10^{5}\right)}$$

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11. Simplify
$$\frac{2^9 \times 3^{13} \times 4^2}{2^6 \times (3^2)^4 \times 4}$$
; then calculate.

(4 marks)

12. Perform the following operations.

a.
$$\frac{1}{2} + \frac{1}{3} \div \frac{1}{6} + \frac{5}{6}$$

b.
$$4 - \left[\left(\frac{3}{8} + \frac{1}{2} \right) \div \frac{1}{4} \right]$$

c.
$$0.48 + 6.30 - 9.77$$

d.
$$2^{-3} \times 2^{5} + \left[3^{2} + \left(\frac{1}{3} \right)^{-3} \right] \div 2^{2}$$

(2 marks)

13. The nearest star, other than the Sun, is about 40 176 000 000 000 km away. Light travels at approximately 1 080 000 000 km/h. Calculate how long it would take light from this star to reach Earth. Express your answer in days.

Value

PART B: PATTERNS AND RELATIONS

30

Answer each of the following questions in the space provided. Clearly show how you arrived at your answers.

(2 marks)

1. Determine the value of n in the equation 5(n+1) = 2(n+4).

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(2 marks)

2. When a number is doubled and decreased by five, the result is less than twenty. Write the inequation that represents this situation.

(2 marks)

3. Solve the equation 5a - 3 = 2a + 3.

(4 marks)

4. Write out the steps that explain how you can use a model (such as cylinders and counters) to illustrate the algebraic solution for 2x-3=-x+6.

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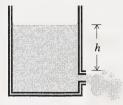
Name of School _____

Date _____

(2 marks)

5. The speed ν (in metres per second) at which liquid discharges from a small hole in a container is given by the formula $\nu = \sqrt{19.6 \, h}$, where h is the height of the liquid above the hole (in metres).

Calculate the height of the liquid above the hole that gives a discharge speed of 5 m/s. Round your answer to the nearest tenth.



(3 marks)

6. One cyclist leaves 2 h before a second cyclist. If the first cyclist travels at a speed of 12 km/h and the second cyclist travels at a speed of 18 km/h, how long does it take the second cyclist to catch up to the first cyclist?

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7. Write an expression that has the following characteristics.

(1 mark)

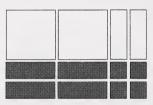
a. a third-degree binomial in two variables

(1 mark)

b. a trinomial of degree 4 in three variables

(1 mark)

8. Write the expression represented by the following group of tiles in simplest form. (**Note:** The white tiles are negative and the black tiles are positive.)

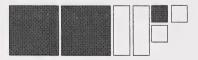


(2 marks)

9. Evaluate the expression $2ab^2 - 4a + 7$ if a = -2 and b = 3.

(1 mark)

10. Draw the tiles that must be added to the following in order to make it equal zero. Use shading to indicate the positive tiles.



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11. Simplify the following:

(1 mark)

a.
$$(2a^3 + 5a^2 - 4) - (a^3 - 2a^2 - 3)$$

(1 mark)

b.
$$\frac{12n^3 - 18n^2 - 9n}{3n}$$

(2 marks)

12. Find the expression for each of the missing dimensions in the following figure if the perimeter is P = 7x - 9.



- 13. Factor each of the following completely.
- (1 mark)
- **a.** $4xy^2 8x$

- (1 mark)
- **b.** $3x^2 9x + 6$
- (1 mark)
- **c.** $a^3 3a^2 10a$
- 14. Multiply the following binomials.
- (1 mark)
- **a.** (n-6)(n+7)

- (1 mark)
- **b.** (4a+3)(4a+3)

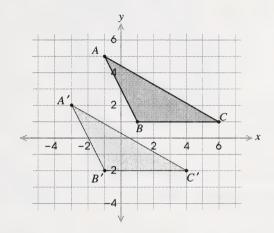
Value

PART C: SHAPE AND SPACE

25

Answer each of the following questions using the space provided. Clearly show how you arrived at your answers.

1. Use the following transformation to answer questions 1.a. and 1.b.



(1 mark)

a. What kind of transformation is being illustrated?

(2 marks)

b. State the mapping rule for the transformation.

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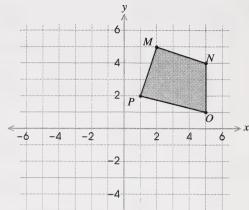
Name of School

Date _____

2. Quadrilateral *MNOP* is given on the grid on the right.

(1 mark)

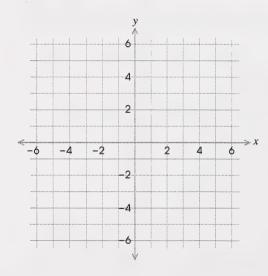
a. Locate a line of reflection such that the image of quadrilateral *MNOP* will be located entirely in the fourth quadrant.



(1 mark)

b. Draw a possible image for quadrilateral *MNOP* (using your line of reflection).

(4 marks) 3. Draw and label a triangle with coordinates A(-4,3), B(0,1), and C(2,4). Draw the resulting image for a rotation of $\triangle ABC$ 180° about the origin. Label the image A'', B'', and C'''.



Name of Student

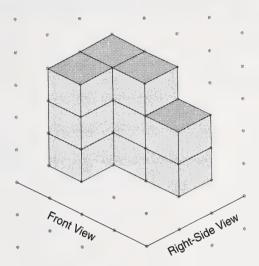
Student I.D. #

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(3 marks)

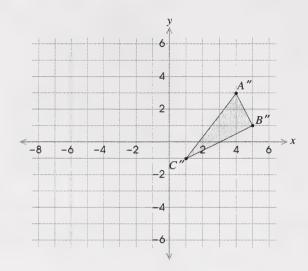
4. Draw and label the front, right-side, and top views from the following three-dimensional drawing.



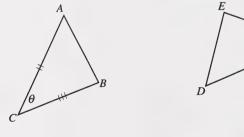
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8	*	٠	*	*	*	*	8	*	•	*	*	8	*	*	8	
	*					*	*		*	*	*	88	•	*	*	

(2 marks)

5. The following triangle was obtained by a translation of 3 units to the right and 2 units up and a reflection about the y-axis. Draw the original position of the triangle and label its coordinates A, B, C; then write the coordinates of A, B, and C.



(2 marks) 6. Is $\triangle ABC \cong \triangle DEF$? If yes, explain how you can tell. If no, give reasons why not.



(3 marks)

7. A wheelchair ramp leading into a public building rises 60 cm over a horizontal distance of 4 m. What is the angle of elevation of the ramp? Round your answer to the nearest tenth of a degree. Show the calculator sequence you used.

(3 marks)

8. A popcorn company is putting out a new line of popcorn that will be available in two different styles of containers: a pyramid-shaped container and a cone-shaped container. The pyramid-shaped container will have square base with each side measuring 15 cm, and the cone-shaped container will have a radius of 7.5 cm. If the height of both containers is 20 cm and they are priced the same, then which container is the better buy for the consumer?

(3 marks)

9. Mildred wants to enclose an area against her 9-m long garage with chain link fence for her dog. If she has 20 m of fencing, what is the maximum area that Mildred can enclose? Assume that the dimensions of the fence must be in whole metres.

16

Value

PART D: PROBABILITY AND STATISTICS

Answer the following questions using the space provided. Clearly show how you arrived at your answers.

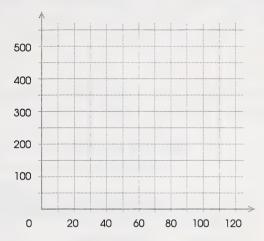
1. A basketball team recorded the number of personal fouls and points for each player. The data is given in the following table.

Player Number	Number of Personal Fouls	Number of Points Scored
1	46	206
3	8	18
4	61	414
6	71	281
11	29	52
14	76	212
22	96	482
28	106	344
34	13	38
40	33	70

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(2 marks)

a. Draw a scatterplot of the data.



(1 mark)

b. What type of relationship does the scatterplot show for the data?

(1 mark)

c. Use a ruler to draw a line of best fit.

(1 mark)

d. Using the line of best fit, predict the number of points a player will score if 60 personal fouls are committed.

(1 mark)

e. How accurate do you think your prediction in question 1.d. is? Explain by referring to the data.

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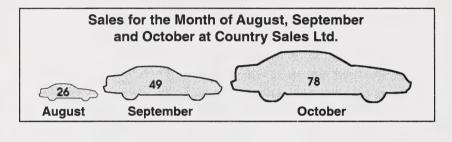
Date _____

(1 mark)

2. Describe the type of relationship you would expect between the number of passes completed by a football team's number one quarterback and the team's final standing at the end of the year.

(3 marks)

3. The following graphic shows the number of cars sold per month for three months at Country Sales Ltd. Explain how the graphic is misleading.



(2 marks)

4. Suppose three quarters are tossed. Calculate the probability that all three will come up heads.

(2 marks)	5.	Eldon collects all the lists of the winning Lotto 6/49 numbers for the past two months. He then draws six numbers at random from all the numbers on these lists. He feels these numbers have a better chance of winning than quick picks (numbers randomly picked by the computer) or any six number sets he would pick. Do you agree or disagree? Explain.
(2 marks)	6.	A medium-sized Canadian city had an average of 21 murders per year for the last ten years. The following year this same city had 11 murders. A newspaper in the city had the following headline.
		January 12, 19xx Page 1
		The Gazette
		CITY'S MURDER RATE DROPS DRAMATICALLY
		Although the headline is correct, explain how it may be misleading.

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	7.	A local weather forecast states that there is a 60% chance of rain on Friday, a 40% chance of Saturday, and a 30% chance on Sunday.
2 marks)		a. Calculate the probability that it will rain on all three days.
2 marks)		b. Explain why the probability calculated in question 7.a. is so low.

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Date _

Name of Student

Name of School _

Formula Sheet

Area

 $A = s^2$ area of a square $A = \ell w$ area of a rectangle $A = \frac{1}{2}bh$ area of a triangle A = bh area of a parallelogram $A = \frac{\left(b_1 + b_2\right)h}{2}$ area of a trapezoid

 $A = \frac{nsa}{2}$ area of a regular polygon

 $A = \pi r^2$ area of a circle

Surface Area

 $SA = 6 s^2$ surface area of a cube SA = 2 B + PH surface area of a rectangular prism SA = 2 B + CH surface area of a cylinder

Volume

V = BH volume of a prism V = BH volume of a cylinder $V = \frac{1}{3}BH$ volume of a cone

 $V = \frac{1}{3}BH$ volume of a pyramid



TEACHER QUESTIONNAIRE FOR MATHEMATICS 9

This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

eacher's Name	Area of Expertise				
chool Name	Date				
esign					
The modules follow a definite systematic of	design. Did you find it easy to follow?				
☐ Yes ☐ No If no, explain.					
Did your observations reveal that the stude	ents found the design easy to follow?				
☐ Yes ☐ No If no, explain.					
Did you find the Learning Facilitator's Ma	nnual helpful?				
☐ Yes ☐ No If no, explain.					
Part of the design involves stating the objeunderstand what they were going to learn?	ectives in student terms. Did you find this helped the students				
☐ Yes ☐ No If no, explain.					

5.		The Learning Facilitator's Manual contains answers to the questions in the Assignment Booklet and a sample test. Did you find these helpful?									
	<u> </u>	Yes	0	No	If no, explain.						
6.		d the Fo		-up Ao No	ctivities prove to be helpful? If no, explain.						
7.		ere stud		motiva	ated to try these Follow-up Activities? If no, give details.						
8.	Suggestions for computer and video activities are included in the course. Were your students able to use these activities?										
	<u> </u>	Yes		No	Comment on the lines below.						
9.	We	ere the	assig	nment	s appropriate?						
	_	Yes		No	If no, give details.						
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1.		ou did Yes	l fax,	did yo No	If no, give details.						

2

Teacher Questionnaire

Mathematics 9

truction	
Did you find the instruction clear?	
☐ Yes ☐ No If no, give details.	
	Terran parties and the second parties are second parties are second parties and the second parties are second parties and the second parties are second parties are second parties are s
Did your observations reveal that the students fo	and the instruction interesting?
☐ Yes ☐ No If no, give details.	
Did you find the instruction adequate?	
☐ Yes ☐ No If no, give details.	
Was the reading level appropriate?	
☐ Yes ☐ No If no, give details.	

5. Was the work load adequate?

☐ Yes ☐ No If no, give details.

6. Was the content accurate and current?

☐ Yes ☐ No If no, give details.

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Use pala menungan ad Lati povildit. E
Advisor was put and special role
Instructional Design and Development Learning Technologies Branch Box 4000 Barrhead, Alberta T7N 1P4
1710 1F4

Mathematics 9 4 Teacher Questionnaire

Note: Please ensure that each of your students has completed and forwarded a copy of the Course Survey.

